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In Tune With The Cosmos: Tuning Theory, Cosmology, And Concepts Of Sound In Early China

Noa Hegesh
University of Pennsylvania, nhegesh@gmail.com

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Abstract

This dissertation explores tuning theories, concepts of sound, and their relation to cosmology in China between the mid-third century B.C.E. and the first century C.E. My overall argument is twofold: First, I argue that to truly understand musical thought in early China, we must realize that it was perceived as a technology. Sound and tuning, especially in a cosmological context, were often discussed in mathematical terms. Second, I argue that for the same reason, we must understand how this technology functioned not only in relation to musical performance per se, but also in what we consider today as non-musical settings, such as mathematical astronomy, the standardization of weights and measures, and divination techniques. Early Chinese authors thought about sound as resonating qi emanating from the cosmos. Its calculation, manipulation, categorization, and measuring were central to the synchronization between the human and cosmic realms.

Part one of the dissertation discusses the gradual introduction of cosmological ideas into existing musical systems around the mid-third century B.C.E. As a result, some texts began discussing sound in numerical terms, as part of theories that aimed to measure the regularities of cosmic processes. Part two explores developments in acoustics and the concept of cosmological sound, through an analysis of a case study from the Western Han: Jing Fang 京房 and his tuning theory, which divided the octave into sixty tuning standards. I also provide an annotated translation of the first section of the "Treatises on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the History of the Later Han. Part Three explores the concept of imperial control in Wang Mang's brief Xin dynasty, examining the connections between sound, metrological practices, and the ideologies and philosophies that provided cosmological meaning to metrological choices. I argue that despite the court's rhetoric of universal standardization, anchored in the dimensions of the Huangzhong pitch pipe, in reality these acts of standardization may not have succeeded far beyond the court's immediate sphere of influence.

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IN TUNE WITH THE COSMOS:

TUNING THEORY, COSMOLOGY, AND CONCEPTS OF SOUND IN EARLY CHINA

Noa Hegesh

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Supervisor of Dissertation

Paul R. Goldin, Professor of East Asian Languages and Civilizations

Graduate Group Chairperson

Christopher P. Atwood, Professor of East Asian Languages and Civilizations

Dissertation Committee:
Nathan Sivin, Emeritus Professor of History and Sociology of Science
Keith Knapp, Professor of Chinese History, The Citadel
Joseph Lam, Professor of Musicology, University of Michigan

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ABSTRACT

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TUNING THEORY, COSMOLOGY, AND CONCEPTS OF SOUND IN EARLY CHINA

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Paul R. Goldin

This dissertation explores tuning theories, concepts of sound, and their relation to cosmology in China between the mid-third century B.C.E. and the first century C.E. My overall argument is twofold: First, I argue that to truly understand musical thought in early China, we must realize that it was perceived as a technology. Sound and tuning, especially in a cosmological context, were often discussed in mathematical terms. Second, I argue that for the same reason, we must understand how this technology functioned not only in relation to musical performance per se, but also in what we consider today as non-musical settings, such as mathematical astronomy, the standardization of weights and measures, and divination techniques. Early Chinese authors thought about sound as resonating *qi* emanating from the cosmos. Its calculation, manipulation, categorization, and measuring were central to the synchronization between the human and cosmic realms.

Part one of the dissertation discusses the gradual introduction of cosmological ideas into existing musical systems around the mid-third century B.C.E. As a result, some texts began discussing sound in numerical terms, as part of theories that aimed to measure the regularities of cosmic processes. Part two explores developments in acoustics and the concept of cosmological sound, through an analysis of a case study from the Western Han:

Jing Fang 京房 and his tuning theory, which divided the octave into sixty tuning standards. I also provide an annotated translation of the first section of the "Treatises on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of the Later Han*. Part Three explores the concept of imperial control in Wang Mang's brief Xin dynasty, examining the connections between sound, metrological practices, and the ideologies and philosophies that provided cosmological meaning to metrological choices. I argue that despite the court's rhetoric of universal standardization, anchored in the dimensions of the *Huangzhong* pitch pipe, in reality these acts of standardization may not have succeeded far beyond the court's immediate sphere of influence.

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INTRODUCTION

This dissertation explores the conceptualization of sound and its relation to cosmology in early China, topics that remain neglected despite the growing interest of Sinologists in musical thought in the last two decades. Whereas most previous scholarship has emphasized the moral and ritual applications of music, I apply my training in music theory and musicology to show how Chinese thinkers, from around the mid-third century B.C.E. to the first century C.E., endeavored to use the calculation and measurement of pitches and scales to synchronize the empire with the cosmos. By analyzing treatises on tuning standards (*lü* 律) and mathematical astronomy (*li* 歷) as well as archeological material, I move beyond the well-studied moral arguments about ritual music, and trace the relation between sound and cosmological ideologies, which culminated between the first century B.C.E., under the Western Han (西漢 206 B.C.E.–8 C.E.), and the first century C.E., with the rise of the Xin dynasty (新 9-23 C.E.).[1]

Sound is the backbone of music, and it had a special status in Chinese culture. It functioned as a central force the relationships between sound and man, and sound and the cosmos. From the late Warring States (475-221 B.C.E.), the numerological representation and measuring of sound became particularly important. Authors began explaining the constructions of scales and interval ratios through numbers and measures in treatises on

[1] I translate *lü* 律 as tuning standard, because of its relative, rather than fixed, function. A *lü* denotes the measuring of sound by using a number. When used in relation to another *lü*, it creates a ratio, which is a music interval. Only when the text ascribes a measure to the *lü* (referring to the length of a string or pipe), it becomes a fixed pitch, and maintains the name *lü*. Pitches within a scale are usually called *yin* 音, and the note names in the five-note scale are *yin* or *sheng* 聲 (at time these terms interchange). Throughout this work I used the term tuning standard most often, but I also use "pitch," when it is certain that the discussion relates to the sonic characteristic or to a recognizable measurement.

tuning standards and mathematical astronomy ("Lüli zhi" 律歷志). Understanding how the early Chinese elite conceptualized sound will help us better understand what they thought about music and the technical ways in which they used sound to fit their political, aesthetic, and cultural needs.

Research on early Chinese music draws on archaeological artifacts, received and excavated texts, as well as the tensions and correspondences among them. Archaeologists, art historians, and musicologists have analyzed the making of instruments, especially bronze bells and stone chimes, and the instrumentation and categorization of early music ensembles. Some of this research has also decoded the musical systems and pitch nomenclature used in different states, mostly through the analysis and corroboration between inscriptions and pitches of the bells and chime sets unearthed in the tomb of Marquess Yi of Zeng 曾侯乙 (d. ca. 433 B.C).[2] Intellectual and cultural historians have studied philosophical argumentation on the function of music as a tool for social and personal cultivation, its close relation to ritual performances, and its role in political culture.[3] Scott Cook's work on the syncretic nature of texts on music is particularly notable.[4]

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[2] Lothar von Falkenhausen, *Suspended Music: Chime-Bells in the Culture of Bronze Age China* (Berkeley: University of California Press, 1993) ; Robert Bagley, "The Prehistory of Chinese Music Theory," *Proceedings of the British Academy* 131 (2005): 41–90. For a general overview and characterization of excavated instruments in early China see: Ingrid Maren Furniss, *Music in Ancient China : An Archaeological and Art Historical Study of Strings, Winds, and Drums during the Eastern Zhou and Han Periods (770 B.C.E. - 220 C.E.)* (Youngstown, NY: Cambria Press, 2008). Some scholars have researched the function and making of other instruments, such as drums and percussive instruments, winds, and strings. Jiang Langchan 蔣郎蟾, *"Zeng Hou Yi mu gu yueqi yanjiu* 曾侯乙墓古樂器研," *Huangzhong* (*Wuhan Music Conservatory*)《黃钟(武汉音乐学院学报)》, no. 4 (1988): 73–84. Laurence Picken, "The Origin of the Short Lute," *The Galpin Society Journal* 8 (March 1, 1955): 32–42.

[3] See, for example, Paul R. Goldin's chapter on music and ritual in the *Xunzi*: Paul Rakita Goldin, *Rituals of the Way: The Philosophy of Xunzi* (Chicago, Ill: Open Court, 1999).

[4] Scott Cook, "Musical Cultivation in the 'Xiu Wen' Chapter of the Shuoyuan," *Dao* 16, no. 3 (September 2017): 389–416. Scott Cook, "The 'Lüshi Chunqiu' and the Resolution of Philosophical Dissonance,"

Like other forms of art, music is sometimes viewed as a cultural product shaped by social settings. Early treatises on the aesthetics of music tend to share many features with later treatises on other art forms, such as painting, and calligraphy, and tend to be quite systematic.[5] Thus, the extraction of the distinctive features of music, which distinguish it as a unique art form, continues to be an important task for modern scholars. The most notable holistic study of early Chinese music remains that of Kenneth J. DeWoskin, who argues for a unity of perspective and terminology in early Chinese texts and does not shy away from analyzing the technical as well as the aesthetical features of music within their historical context. To him, musical aesthetics in early China was the way man responded to order and pattern in the cosmos.[6]

Musical performances included not only performers, but also an affected audience. Thus the study of music within a ritual context extended also to the realm of the sensorial and corporal experience.[7] Western Han histories such as the *Shiji* 史記 and political-philosophical texts such as the *Huainanzi* 淮南子 advocate different, at times contradictory,

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Harvard Journal of Asiatic Studies 62, no. 2 (December 2002): 307–45. Scott B. Cook, "Unity and Diversity in the Musical Thought of Warring States China." (University of Michigan, 1995).

[5] For studies on the aesthetics of calligraphy and painting see William Acker, *Some T'ang and Pre-T'ang Texts on Chinese Painting* (Leiden: Brill, 1954).

[6] Kenneth J DeWoskin, *A Song for One or Two: Music and the Concept of Art in Early China* (Ann Arbor: Center for Chinese Studies, University of Michigan, 1982).

[7] For an in-depth analysis of ritual settings see: Scott Cook, "Xun Zi on Ritual and Music," *Monumenta Serica* 45 (1997): 1–38. And Ori Tavor, "Embodying the Way: Bio-Spiritual Practices and Ritual Theories in Early and Medieval China" (University of Pennsylvania, 2012). For a discussion on the senses in early Chinese philosophical texts see Jane Geaney, *On the Epistemology of the Senses in Early Chinese Thought*, Monograph / Society for Asian and Comparative Philosophy, no. 19 (Honolulu: University of Hawaii Press, 2002). And for an analysis on senses and emotions of the sage in the *Huainanzi* see Griet Vankeerberghen, "Emotions and the Actions of the Sage: Recommendations for an Orderly Heart in the 'Huainanzi,'" *Philosophy East and West* 45, no. 4 (October 1, 1995): 527–44.

approaches to the regulation of the senses and thus to the function of man-made music versus natural and cosmological sounds on the individual and society. Thus, despite the common goal of cultivating the ruler and society, and the use of similar terminology and music theory, the different approaches to the effects of sound on the senses in texts on ritual performance should be studied further.

Recently, there has been a growing interest among scholars in the role of cosmology in the conceptualization of music in late Warring States and early imperial China and its use of terminology such as *yin* 陰 and *yang* 陽, *qi* 氣, harmony (*he* 和), and sympathetic resonance (*gan ying* 感應), among others.[8] Erica Brindley's recent study argues that music "stood out from other traditional, cultural, and man-made practices. It took on a meaning as that which was 'natural' or 'intrinsic' to the subtlest aspects of the cosmos."[9] Her research analyzes texts down to 100 B.C.E., focusing on musical and cosmological terminology, and the array of theories that connected music and the cosmos via rulership. Previous Western scholarship on this subject focused on either its philosophical or technical aspects, but rarely both.[10] Brindley's research belongs to the former, as she notes that she "[does] not engage many of the mathematical, harmonic relationships designated by theory that emerged in

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[8] More research should be done on the term *xie* 協, which by the Han denoted a cosmic correspondence, especially regarding sound.

[9] Erica Brindley, *Music, Cosmology, and the Politics of Harmony in Early China* (Albany: State University of New York Press, 2012), 63.

[10] For a musical analysis of Jing Fang's sixty modes see: McErnest G. McClain and Ming Shui Hung, "Chinese Cyclic Tunings in Late Antiquity," *Ethnomusicology* 23, no. 2 (May 1, 1979): 205–24. Also see Marc Kalinowski, "Musique et Harmonie Calendaire à La Fin Des Royaumes Combattants: Les Livres Des Jours de Fangmatan (239 Avant J.-C.)," *Etudes Chinoises* 30 (2011): 99–138. On the philosophical aspect, Brindley's book, mentioned above, is still the best source.

tandem with notions of cosmic music."[11] In this sense, the present project complements her work. Early Chinese conceptualization of sound — especially when associated with cosmology — was a mathematical art form, and temperament is indispensable to its understanding. We know this because early Chinese histories make sure not to restrict this discussion to a philosophical level. They incorporate calculations of intervals, relate pitches to the seasons and treatises dedicated to tuning standards and mathematical astronomy. Most importantly, by the first century B.C.E., mathematical calculations of musical modes and pitches had become entwined and even led by cosmological practices such as traditions of hexagram interpretation and mathematical astronomy.

Scholars who research sound or music in early China face the grand problem of decontextualization. As Ann E. Lucas defines the problems historical ethnomusicologists face in researching oral histories in modern libraries and archives: "…they exist within contemporary realities of both scholarship and music that inescapably decontextualize the musical past… It is the difficulty of history in most cases: inevitably, context is lacking to one extent or another."[12] A similar problem affects my own research. The official histories of ancient China are our libraries and archives. They provide a polished and idealized presentation of practices, written according to the preferences of the editor, compiler, and author. Our biggest chance at contextualization comes from archeological evidence and scrupulous textual comparisons. Still, the problem of corroborating theory with records of practice appears and reappears in this dissertation because the latter are lacking. In the case

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[11] Brindley, *Music, Cosmology, and the Politics of Harmony in Early China*, 12.

[12] Ann E. Lucas, "Ancient Music, Modern Myth: Persian Music and the Pursuit of Methodology in Historical Ethnomusicology," in *Theory and Method in Historical Ethnomusicology*, ed. Jonathan McCollum and David G. Herbert (London: Lexington Books, 2014), 176.

of early China, we have no compositions or direct evidence that shows how scales and modes were put to use. But the texts do assert a connection between theory and practice, at least in a ritual setting.

This dissertation is divided into three large parts, chronologically arranged from the Warring States to the Western Han and the Xin dynasty.

The first part of this work is titled "The Road to Cosmological Sound." This chapter traces a path by which an existing tradition of music theory and a tradition of cosmology conflated. By the Han, the merging of these two systems created a cosmological reality that placed sound at its core, and a musical system based on cosmological notions. I begin by showing that fully operable musical systems, in use around the mid-fifth century B.C.E., show no direct relation to cosmological thought.

Next, I show how authors began incorporating cosmological principles into their discussions of the musical system, around the fourth and third centuries B.C.E. This incorporation is evident in mythologies on the creation of tuning standards and in a description of the scales used in ritual music in The Grand Minister of Music (*Da Siyue* 大司樂) section in the "Spring Offices, [Domain of the] Patriarch of Ancestral Air" (*Chun Guan Zong Bo* 春官宗伯) of the *Rites of Zhou* (*Zhouli* 周禮). This section offers detailed information about how tuning standards were to be used in ritual performance and in musical education. Finally, I turn to divination in the mid-third century B.C.E. Scroll B of the recently excavated daybooks (*rishu* 日書) from Fangmatan attests to the use of a fully developed complex numerological representation of tuning standards outside the musical

realm. This system was an independent tool for assessing cosmic and geomantic conditions in divination, which did not require instruments or audible sound.

Part Two of this work, titled "Mind the Gap: Jing Fang and his theory of sixty tuning standards in the Western Han," analyzes a case study in the development of cosmological sound in the first century B.C.E. I argue that important advances in musical temperament were propelled by cosmological frameworks. Cosmological thought enabled literati to construct a rich and complex world of cosmic sound in the offices of astrology and astronomy. Section 1 introduces the biography and accomplishments of Jing Fang 京房 (78-37 B.C.E.). Section 2 is an annotated translation of the first section of the "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of the Later Han* (*Houhan shu* 後漢書), which presents Jing Fang's theory. Finally, section 3 is an analysis of the acoustical and cosmological aspects of the treatise, including Jing Fang's calculations of the sixty tuning standards, his motivation for reducing the Pythagorean comma, and the underlying cosmological concepts that drove his innovation.

Part Three is titled "The Fantasy of Control: the function of *Huangzhong* in Liu Xin's theory of weights and measures." This chapter explores the concept of imperial control in the Xin dynasty (9-23 C.E.) by examining the connections among sound, metrological practices, and metrosophical theory – i.e. the ideologies and philosophies that provided cosmological meaning to metrological choices. I argue that this activity, like the use of tuning standards in divination, did not require the use of audible sound. The dimensions of the *Huangzhong* pitch pipe, as set by the Xin administration, were the only consideration in the act of standardizing weights and measures. I also argue that this act of standardization was a

display of power by Wang Mang in his assertion of his cosmic mandate. The *Huangzhong* tuning standard and the methods used to measure it correctly were the symbol synchronization with the cosmos. Its use in astronomy and in weights and measures signaled the ultimate role of the emperor as the nexus between Heaven and earth. Lastly, I argue that despite the imperial demand for precision in the calculation of length, volume, and weight measures, new archeological evidence shows that communities in different locations across the territory continued using divergent standards for unit measures. These measures tended to vary quite a bit from the imperial standard.

I begin the chapter by discussing the development of the idea that sound could function as a standard for weights and measures. Next, I analyze Liu Xin's "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of the Former Han* (*Han shu* 漢書). After analyzing the requirements set out by Liu Xin, I treat each of them individually by examining archeological artifacts that embody weights and measures. Finally, I examine Liu Xin's method of using grains of millet to measure the dimensions of *Huangzhong*.

My overall argument in this dissertation is twofold. First, I argue that in order to truly understand musical thought in early China we must familiarize ourselves with its conceptualization as technology. This familiarity should include music theory, acoustic endeavors, gaps in our knowledge of theory versus practice, and the gradual weaving of cosmological and numerological ideas onto musical actions such as selecting scales, ritual tuning of instruments, and aspects in music temperament. Second, I argue that for the same reason, we must understand how this technology functioned not only in relation to musical performance per se, but also in what we consider today as non-musical settings (though a

clear dichotomy did not exist at the time). The dimensions of a pitch pipe were used to convey imperial standards for weights and measures, without using its sound. The names and ratios of tuning standards were used in hexagram divination without having to be heard. The basis of sound became grounded in numbers, and to speak of sound in numbers was one and the same. These began appearing from around the third century B.C.E. onward, and relied on the idea that sound was measurable resonating *qi* emanating from the cosmos.

The three main categories of investigation are the use of tuning standards in astronomy; the use of tuning standards in hexagram divination; and the significance of the *Huangzhong* pitch pipe in the emperor's act of standardizing of weights and measures. The role of the ruler and his officials was to detect, measure, calculate, and ritualize sound in order to keep the realms of heaven and earth synchronized. Treatises that focus on ritual musical performances rarely allow us a glimpse into these dimensions of sound conceptualization. Without them, our understanding of the musical world of early China remains incomplete.

Part One

THE ROAD TO COSMOLOGICAL SOUND: FROM MUSIC TO RATIOS

Of the received textual sources predating the fourth century, the *Book of Odes* 詩經 is replete with musical references and attitudes toward musical styles, especially in ritual context.[13] The *Book of Odes* is a collection of texts that were musically performed.[14] Music was far from a cacophony of sounds. Musicians were instructed to follow aesthetic parameters that made their playing together harmonious (*he* 和 or *tong* 同). The available data do not indicate any detailed rules or terminology for music theory, or even the components of the musical system musicians used, so we do not know how this harmony was achieved in practice. The closest thing to it was a description of timbre and tuning in the *Book of Odes* in the ode "Pealing the Bells" (*Gu zhong* 鼓鍾), in the Lesser Elegantiae (*Xiao ya* 小雅, sometimes translated as Minor Court Songs) section, which, quite fittingly, was said to have been written by music masters:[15]

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[13] The poems in the *Books of Odes* are dated some time between c. 1000 to c. 600 B.C.E. Michael Loewe, *Early Chinese Texts : A Bibliographical Guide* (Berkeley, CA: Society for the Study of Early China : Institute of East Asian Studies, University of California, Berkeley, 1993), 415.

[14] Laurence Picken, "The Shapes of the Shi Jing Song Texts and Their Musical Implications," *Musica Asiatica* 1 (1977): 85–109. Michael Nylan, *The Five "Confucian" Classics* (New Haven: Yale University Press, 2001), 72–73.

[15] Nylan, *The Five "Confucian" Classics*, 73. DeWoskin also translates the title as a verb-object, "Hammering the Bells" (DeWoskin, *A Song for One or Two*, 21. n.5). The first line of this section alone shows the character 鼓 *gu* used three times as a verb for playing different instruments. Moreover, the only word used in relation to a drum in this Ode is 伐 (*fa*), also used in the verb-object: 伐鼛 (*fa gao*) – to beat the drum.

鼓鍾欽欽、鼓瑟鼓琴。	Pealing bells "qin-qin," plucking *se*, strumming *qin*.
笙磬同音。	The *sheng* and chimes play corresponding tones.
以雅以南、以籥不僭。	In the Elegant and Southern [odes], none surpass the *yao* flutes.[16]

The full ode reflects on how a musical performance stirs the emotions of the listener, in this case a woman, who expresses her anxiety for the fate of her beloved who is on a mission in the east, while extolling his character. The ensemble instruments are introduced gradually, beginning with the bells, adding the drums, and finally describing the performance, adding the strings, chimes, mouth organ, and pipes. The term "corresponding tones" (*tong yin* 同音) is an indicator of an aesthetic guideline for ensemble playing. It may refer to the ability of musicians to sound together, but it may also refer to an either structural or tonal aesthetic. There is nothing further to extract regarding the music system in use.[17] Laurence Picken is perhaps the only Western scholar who attempted to analyze the structural and rhythmic patterns in the *Book of Odes*, but even he does not claim any knowledge of music theory or

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[16] There is no agreement on the translation of "以籥不僭." Karlgren translates it as "the flutes play without error," but gives no explanation for his choice, except noting that the ritual dancers were those who played the flute during ritual performance (Bernhard Karlgren, *Book of Odes: Chinese Text, Transcription and Translation* (Stockholm: The Museum of Far Eastern Antiquities, 1950), 160). DeWoskin translates "the short flute holds its own."

[17] Brindley has dedicated a section to the relation between music, emotions, and morality, as in the composition of songs, especially as a public expression of the innermost emotions of the Gentleman (Brindley 2012, 92-4). I focus on the indication of a musical system or musical temperament within the wide subject of music, and so I do not elaborate on the large topic of music making in the *Book of Odes*.

DeWoskin points out that the Elegant (Ya) and Southern (Nan) Odes possibly refer to the performance section carrying the same titles in the *Book of Odes*. Brindley gives two additional short examples of self-referencing in the Odes, which tell the reader/listener about the conditions that brought about their composition (Brindley, 93.). This device is not limited to musical references, and was used throughout the *Book of Odes*.

the details of the aesthetical guidelines of the times.[18] Music systems existed before being mentioned in textual sources, and their development, at first, had nothing to do with cosmological ideas, which had not yet emerged. Analysis of excavated *xun* 塤 (clay ocarinas) has shown that the five-note scale (and its derivative variations) existed as early as the Shang.[19] Bagley discusses an assembled bell set that produces six consecutive semitones (and additional bells that did not fit into the chromatic scheme), creating a chromatic hexachord dating to the twelfth or eleventh century. The analysis of inscriptions on bell and chime sets excavated from the tomb of Marquess Yi of Zeng (Zeng Hou Yi 曾侯乙), as well as an analysis of the pitches produced by the bells, have also shown that by the mid fifth century B.C.E., China enjoyed several elaborate and complex musical systems that placed great importance on the organization of named musical intervals, namely a twelve tone basic musical scale, in which each pitch could function as a fundamental for the construction of a scale using a selection from the other pitches.[20]

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[18] Lewis first discusses winds in non-cosmological aspects, referring to the "Feng" 風 section of the *Shijing*, which was sung with musical accompaniment, thus he defined "Feng" as a type of music or simply a "tune." His following example taken from the *Zhuangzi*, however, ignores the purpose of the author to distance himself from man-made music and arguing music to strive to imitate the natural sounds of "bellow, gasp, shout, scream, moan, and howl" of winds through the openings slits and slots of trees in nature. These refer to and endorse unregulated sound, what in the Western Han, perhaps, Jing Fang would term "sounds of the grass and trees," which cannot be harmonized. See the translation of this section in Part Two (p. 75). Lewis 1990, 218–19.

[19] DeWoskin, *A Song for One or Two*, 44 n.3.

[20] Lothar von Falkenhausen, "On the Early Development of Chinese Musical Theory: The Rise of Pitch-Standards," *Journal of the American Oriental Society* 112, no. 3 (1992): 433–39. The inscriptions on the bell set show the Zeng musical system, while the inscriptions on the chime set show that of the neighboring state of Chu. Falkenhausen has also shown that the Zeng bell set nomenclature could have been influenced by other earlier systems.

In this chapter I argue for the following four points:

- Cosmology and the music system did not develop simultaneously. Many features of an existing musical system or systems existed with no direct evidence for cosmological underpinning. From around the fourth century B.C.E., a musical system was placed within a cosmological context. By the mid-third century B.C.E., the mathematical expression of tuning served to express cosmological harmony. Thus, the mathematical expression of tuning standards became a divinatory tool and they became related to hexagram interpretation. Part Two of the dissertation shows a reversal of this process during the Han dynasty, in which the will to make the musical system adhere to cosmological principles propelled acoustic developments that aimed to reflect a complete understanding of the way sound functioned in the cosmos.

- Early mythology on the generation of the tuning standards is cosmological in nature. Tuning measured the cosmic state.

- The *Rites of Zhou* (*Zhouli* 周禮) ateest to the incorporation of cosmological principles in the selection of scales used in the performance of ritual music and songs.

- The recently excavated daybooks of Fangmatan attest to the use of a fully developed complex numerological representation of tuning standards outside the musical realm in the mid-third century B.C.E.

The Zeng Bells

In 1978, archeologists of Hubei province 湖北 excavated the tomb of Marquess Yi 侯乙, a ruler of a state called Zeng 曾, who died c. 433 B.C.E. The excavation revealed, for the first time, the richness of musical practice and theory at the time. The tomb chambers contained 125 musical instruments, some of which belonged to small string and wind chamber ensembles, and others to a large ritual orchestra. The large ritual orchestra unearthed in the central chamber of the tomb included strings (seven *se* 瑟 zithers), mouth organs (four mouth organs *sheng* 笙), winds (two bamboo panpipes *xiao* 簫, two transverse closed end bamboo flutes *chi* 篪), three drums (suspended *jian gu* 建鼓, and hand held *xiao gu* 小鼓), thirty-two suspended stone chimes (*bianqing* 編磬), and a set of sixty-five suspended two-toned almond-shaped bells (*bianzhong* 編鐘).[21] The string, wind, reed, and small drums were likely set against the eastern wall of the chamber, although some had been carried away by accumulated water in the tomb, together with the large drum, set at the southern part of the wall. The bell set and L-shaped suspension stand were set against the western and southern chamber walls. A set of stone chimes and their stand were set against the northern wall. This was likely the ritual setting of a large orchestra.

In the east chamber (*dongshi* 東室) where the coffin was kept, the instruments of a smaller ten piece ensemble were also found. The ensemble contained two eighteen-reed

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[21] *Hubei sheng bowuguan* and *Zhongguo shehui kexueyuan. Kaogu yanjiusuo* 湖北省博物館, 中國社會科學院中考古研究所編輯, eds., *Zeng Hou Yi mu* 曾侯乙墓, 2 vols. (Beijing: *Wenwu chubanshe* 文物出版社, 1989).

sheng, five *se*, a single ten string *qin* zither (琴), a single tuner (*yunzhong* 均鐘),[22] and a single drum.[23] In addition to instruments, the tomb also contained a supply of replacement parts. The main chamber and the east chamber contained a square bamboo basket full of V-shaped movable bridges (*zhu* 柱) for the *se* zithers.[24] Also, the north chamber (*beishi* 北室) contained an empty stone chest for the storing of chime stones. These supplies attest to

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[22] *Yun* 均 means equal division, regulated, or standardized. It seems that in a musical context this character should be pronounced *yun* and not *jun*, although this is not always the case. As a variant of 鈞, it may be pronounced *jun* (DeWoskin, *A Song for One or Two*, 47). On the different pronounciation of the charachter 均 see Paul R. Goldin, "Two Notes on Xie He's 謝赫 'Six Criteria' (Liufa 六法), Aided by Digital Databases," Forthcoming.

The Bell Tuner was not a bell and it was not used for performance purposes, but as a tone marker, to which the orchestra could tune. The exact categorization of the instrument found in Marquess Yi's tomb, however, has been debated. Li Chunyi identifies it as a *yunzhong*, but prefers to call it a Five String [zither] (*wuxian* 五弦). The *yun* Li refers to is as an instrument mentioned in the "Discourses of Zhou"("Zhouyu" 周語) chapter in the *Discourses of State* (*Guoyu* 國語). He cites Wei Zhao's 韋昭 (204-273 C.E.) commentary: "The *yun* is a wooden bell standardizer. Its length is seven *chi*. It has strings tied to it in order to standardize bells. It determines the size and pitch (*qingzhuo* 清濁) of a bell. The *Da Yu* Music Master in the Han had it." 均者。均鍾木。長七尺。有弦繫之以均鍾者。度鍾大小清濁 也。漢大予樂官有之。(see more on the context of this *Guoyu* passage and commentary on p. 128). In my view, it is unclear whether the instrument in the tomb is the same as the instrument the *Guoyu* describes, and its name in excavation reports and such reflects false certainty. For further information on this debate and classification see Li Chunyi 李純一, *Zhongguo Shanggu Chutu Yueqi Zonglun* 中國上古出土樂器綜論 (Beijing: *Wenwu chubanshe* 文物出版社, 1996), 460. Furniss 2008, 71-72; Huang Xiangpeng 黃翔鵬 and Wang Zichu 王子初, *Zhongguo Yinyue Wenwu Daxi* 中国音乐文物大系, vol. Hubei 湖北 (Zhengzhou shi 郑州市: *Daxiang chubanshe* 大象出版社, 1999), 281–83.

[23] Huang Xiangpeng 黃翔鵬 and Wang Zichu 王子初, *Zhongguo Yinyue Wenwu Daxi* 中国音乐文物大系, Hubei 湖北:196–197. Ingrid Furniss provides eight other examples of wind, strings, and/or drum ensembles dating to the Warring States periods (she does not mention the Marquis's eastern chamber ensemble in this list, but discusses it in another chapter). The number of instruments in this type of small chamber ensemble varies, but they were not uncommon in tombs of the wealthy elite of this period. Furniss 2008, 140-151. Furniss also suggests that this type of ensemble shows the dominant role of strings in private chamber ensembles. I tend to think that two eighteen reed *sheng* would easily be as loud as five or six *se* or *qin* and it was probably the blending of these sounds that dominated the ensemble. If we could know for certain that *sheng* was not used for melodic lines, there would be place to argue over dominance. Furniss, 224.

[24] The *Zhu* (柱) are V-shaped wooden pieces placed under the strings of the *se*. Their placement determines the nodal point at one end of a vibrating string, in effect determining its length and the pitch it produces. For a general explanation on the parts of the *se* see "Gu Yinjie 古音階," *Zhongguo Yinyue Cidian* 中国音乐词典 (*Renmin yinyue chubanshe* 人民音乐出版社, 1984), 335. For a more detailed analysis and illustration see Huang Xiangpeng 黃翔鵬 and Wang Zichu 王子初, *Zhongguo Yinyue Wenwu Daxi* 中国音乐文物大系, Hubei 湖北:276–277.

either a belief or a symbolic gesture that the instruments were meant to be tuned and played. Their arrangement is probably the same as that of a live ensemble from that period.

Our knowledge of at least two musical systems used in this period comes from the inscriptions engraved on the bell and chime sets of the large orchestra in the main chamber.[25] Of the instruments found in the tomb, the bells are the only ones that survived in playable condition, allowing exact pitch measurement and analysis of the results in comparison with the musical systems the inscriptions describe. Lothar von Falkenhausen analyzed and explained the music system of the Zeng bell set and that of the chime set in its complexity in his 1994 book.[26] The Zeng bell inscriptions reveal a musical system based on successive major thirds (an interval between two pitches equaling two whole tones).[27] The

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[25] Falkenhausen argues that "Close analysis reveals that the Zeng inscriptions synthesize fragments of a number of different but related musical systems." Falkenhausen, *Suspended Music*, 283.

[26] Robert Bagley has also written about the Zeng system, focusing on the upper bell tier, inscriptions on the chime stones and their storing box. In his article, he argues that "Pentatonic scales are selections from the chromatic scale." (p. 64) and that "The chime-stone inscriptions do not seem to have been written by a Zeng theorist. Instead of the Zeng pitch standards they use the standards of a neighboring state, the state of Chu, and Chu theorists had twelve pitch standards spaced at semitone intervals" (p. 70). He also hypothesized that assembled sets of signaling bells were a precursor to tuned bell sets and that when casting a tuned set "bronze founders learned to do it by copying assembled sets." (p. 76) See Bagley, "The Prehistory of Chinese Music Theory." Many of these ideas are mentioned in Falkenhausen's book, Bagley brings them together more concisely and clearly (with no credit to Falkenhausen's main body of work).

I disagree with Bagley's last argument in the article, which suggests that string players discovered their ability to transpose pieces beginning at each of the twelve tuning standards through the way bell sets were organized. He says: "I can imagine a musician putting those twelve pitches on a zither and verifying for himself that each string in turn could be taken as the starting point for a pentatonic scale." Strings are far easier to tune than bells, and transpositions can be done quite easily, in any kind of music, ritual or folk, even by simply using one's ears, especially when accompanying vocals (which often require the player to use a key that suits the singer's range). String players would have used the twelve tuning standards as a base and transposed melodies at will long before any bell set would have been able to set the system for it. They were likely able to use twelve pitches in various additional ways that a fixed bell set could not (ornamentation, or melodies using different scale divisions are but two options). Setting down terms and rules for the musical system may have come from the use of fixed bell sets, and string players may have tuned their instrument to match the fundamental produced by a fixed bell (or an entire scale of a bell set), but the ability to comprehend and use transpositions surely did not start there.

[27] Falkenhausen, *Suspended Music*, 296. A whole tone is "The interval equal to the sum of two semitones and hence referred to as a 'whole tone', usually perceived as a major 2nd; in equal temperament, the sixth part of an octave. It and the semitone are the intervals by which conjunct motion in a part or voice is

two most important terms in this system are the suffixes *jue* 角: raise by an interval of a Major third (two tones), and the suffix *zeng* 曾 (may have stood for 增 augment): raise by an interval of two Major thirds (four tones).

In later textual sources, the pattern known as the five notes (*wu sheng* 五聲) fixed *jue* 角 in the third note of the scale, a Major third interval from the first note: *gong, shang, jue, zhi, yu* (If we choose to begin with C, *jue* would be E and the pentatonic pattern would be C, D, E, G, A).[28] This fixed position probably developed from its role in systems like that of Zeng, where it is only used as an added suffix to any note in the four note scale: *gong, shang, zhi, yu* (C, D, G, A).[29] This is a system based on extensions, in which note names can stand alone (e.g *gong* 宮) or come with a suffix (*gongjue* 宮角, *gongzeng* 宮曾).[30] Together they create a gamut of twelve chromatic notes. If we choose middle C as *gong*, then *gongjue* would be E, and *gongzeng* would be G sharp. In the following table I show how to produce a twelve note (unfixed) chromatic scale using the four notes and their intervallic suffixes within a

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generated." William Drabkin, "Tone (Iv)," *Grove Music Online. Oxford Music Online* (Oxford University Press, n.d.), accessed November 29, 2016.

[28] This is also called an anhemitonic pentatonic scale, which is a five-note scale that does not include semitones.

Other intervallic affixes also exist in the Zeng inscriptions. These include *bian* 弁, which lowers the pitch by a semitone, and the prefix *zhuo* 濁 in the chimestones inscriptions and in the names of six tuning standards, set one tone apart, on the bells, which does the same. The Zeng nomenclature seen in the inscriptions seems to reveal, as Falkenhausen suggests, fragments of several related systems. Additionally, both Bagley and Falkenhausen show how the use of the prefix *zhuo* links the Zeng system to that of the state of Chu based on an excavated set of pitch pipes dated to the fourth century B.C.E., in Yutaishan, Jiangling (Hubei), which also include *zhuo* as a prefix. For the use of *zhuo* on the Zeng bells in the indigenous Zeng nomenclature see Falkenhausen, *Suspended Music*, 289.

[29] Although the Zeng inscriptions provide the earliest example of the five note 五聲 nomenclature, used in a scheme based on intervals of Major thirds, "these names must have had considerably earlier origins." Falkenhausen, 285.

[30] Falkenhausen, 284.

single octave. The notes marked in red are the basic four note scale from which to begin, and the bottom row gives the full chromatic series:

→ Each of the four main notes: 宮，商，徵，羽 and their two "inversions" within a single octave:

GONG 宮	C *Gong* 宮				E *Gongjue* 宮角				G# Gongzeng 宮曾				B# (≅C
YU 羽		C# Yujue 羽角				E#(≅F) Yuzeng 羽曾				A Yu 羽			
SHANG 商			D Shang 商				F# Shangjue 商角				A# Shangzeng 商曾		
ZHI 徵				D# Zhizeng 徵曾				G Zhi 徵				B Zhijue 徵角	
	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C

This pattern, or part of it, can then be applied to the tuning standards (*lü* 律), which are also inscribed on the Zeng bells. In essence, the tuning standards determine the key in which a scale is constructed, while the note names denote positions within that key (first, second, third, fourth). For example: *gong* of *Suibin* 妥賓之宮, means that the pitch of this bell sits in the first position on a scale that begins on the *Ruibin* pitch (meaning the *Ruibin* key). But the very same bell can also act as *shangjue* of *Huangzhong* 黃鐘之商角, meaning that its pitch sits in the third position of a scale that begins on the *Huangzhong* pitch (the *Huangzhong* key). Although the pitch remains the same, the inscriptions guide us to the possibilities for using it in different positions in different keys.

The nomenclature of the tuning standards in the Zeng inscriptions also uses names from several locations under the Zhou. Of the twenty-one names given, only nine are indigenous to the Zeng state, and in some cases the inscription includes additional references

to the equivalent of a tuning standard in a different state (for example: "As to *Suibin*, in Chu it is called *Pinghuang*… 妥賓之在楚號為坪皇…").[31]

Briefly stated, the Zeng inscriptions presents a set of twelve tuning standards (which are not, taken together, perceived as a chromatic scale in their own right), and additional tuning standard names for octavic equivalences and for nomenclature originating in other states. The Zeng nomenclature most resembles the tuning standards lists in the "Spring Offices, [Domain of the] Patriarch of Ancestral Affairs" (*Chun Guan Zong Bo* 春官宗伯) Chapter in the *Rites of Zhou*, which will be discussed later in this chapter, as well as the "Discourses of Zhou" (*Zhouyu* 周語) section of the *Discourses of the States* (*Guoyu* 國語).[32] Most important is the likelihood that they were conceived not as a single set of twelve, but as six core tuning standards, and six secondary ones.[33] This is apparent in textual divisions of the tuning standards to *yin* and *yang*, *lü* 律 and *jian* 間, *lü* 律 and *tong* 同, or *lü* 律 and *lü* 呂. Also, on numerous occasions references to the musical system discuss the "six tuning standards" (*liu lü* 六律) rather than twelve, clearly referring to the six core tuning standards, but likely addressing the system as a whole, including the secondary ones.

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[31] Falkenhausen, 289. The nomenclature of the tuning standards is also a complex subject, which I chose not to elaborate on here. In essence, only nine of the tuning standard names seem to belong to the indigenous Zeng system. Five of the names (*Guxian, Suibin, Wuyi, Huangzhong, Taicou*) correspond to the traditional names (those who appear in textual sources), while three others (*Yingzhong, Xuanzhong, and Hanyin*), have their textual equivalents in name but not in scale position. For example, *Yingzhong* and *Xuanzhong* are octavic corresponders to *Huangzhong* and *Guxian* in the Zeng inscriptions, and in textual sources they are part of the six secondary tuning standards in the set of twelve. Falkenhausen 1993, 286, 288-289

[32] Falkenhausen compares these variations and provides a useful table, which also includes the names of the twelve tuning standards in their standardized version from the Han.

[33] Falkenhausen, *Suspended Music*, 288.

The "Discourses of Zhou"[34] is the earliest textual evidence of twelve tuning standards and their division into *yin* and *yang*. A long section concerning tuning provides two accounts in which Ruler Jing of Zhou 景王 (r. 544–520 B.C.E.) wished to cast a *Wuyi* 無射 bell. In the first, Duke Mu of Shan 單穆公 reprimanded Ruler Jing for wishing to cast a bell in the pitch of *Wuyi* and intending to "make it into *Dalin*" (*er wei zhi dalin* 而為之大林). In the second account, Jing was about to cast a *Wuyi* bell and asked Ling Zhoujiu 伶州鳩 about tuning.[35] It is in this section that the names of the tuning standards appear. The text does not list them as a single ascending scale, but as two sets, which are not organized by order of generation.[36] Six tuning standards (i.e *Huangzhong*, *Taicou*, *Guxi*, *Ruibin*, *Yize*, and *Wuyi*) are categorized as *lü* 律, and the remaining six (i.e *Dalü*, *Jiazghong*, *Zhonglü*, *Linzhong*,

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[34] The *Guoyu* may have been edited sometime after c.425 B.C.E., but includes some sections dating to the Warring States. David R. Knechtges and Taiping Chang, eds., *Ancient and Early Medieval Chinese Literature: A Reference Guide*, vol. 1, Handbook of Oriental Studies. Section Four, China, Handbuch Der Orientalistik, v. 25-1--25-4 (Leiden [Netherlands] ; Boston: Brill, 2010), 311. ; see also William G. Boltz, "Kuo Yü 國語," in *Early Chinese Texts: A Bibliographical Guide*, ed. Michael Loewe (Berkeley, CA: Society for the Study of Early China : Institute of East Asian Studies, University of California, Berkeley, 1993), 263–68.

[35] For a more detailed analysis and translation of these accounts, and the possible meaning of "making it into *Dalin*," see Part Three, p. 125.

[36] Kalinowski also uses the *Rites of Zhou* (*Zhoulii* 週禮) chapter "Spring Offices, [domain of the] patriarch of ancestral air" (*Chun Guan Zong Bo* 春官宗伯) as a mid-third century B.C.E. text. William G. Boltz dates it only as a pre-Han text, probably not later than the second century B.C.E. See William G. Boltz, "Chou Li 周禮," in *Early Chinese Texts: A Bibliographical Guide*, ed. Michael Loewe (Berkeley, CA: Society for the Study of Early China : Institute of East Asian Studies, University of California, Berkeley, 1993), 24–32.

The "Da Siyue" section in the *Rites of Zhou* also uses slightly different terminology and classifies the six *yin* tuning standards as "corresponding" (*tong* 同). See also Kalinowski, "Musique et Harmonie Calendaire," 107. The two sets are still also classified in that section as *yang* sound (*yang sheng* 陽聲) and *yin* sounds (*yin sheng* 陰聲), and also as *Lülü* 律呂. Thus, the familiar dichotomy remains. Two of the tuning standards in the set, *Hanzhong* 函鐘 and *Xiaolü* 小呂, do not appear in other sources. Zhengxuan glosses *Hanzhong* as *linzhong* 林種, and *Xiaolü* as *Zhonglü* 中呂, which are their equivalent names in later treatises. Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, eds., *Zhouli Zhengyi* 周禮正義, vol. 7, *Shisan jing qing ren zhushu* (Beijing: Zhonghua shuju 中華書局, 1987), 1757.

Nanlü, and *Yingzhong*) as "interstitial" (*jian* 間). The merging of these sets into a single concept known as the twelve tuning standards (*shier lü* 十二律), happened some time during the middle of the Warring States.[37]

In summary, the Zeng inscriptions reveal a fully operable complex musical theory system that paid great attention to tuning standards, pitch measurements, intervals, and transposition options, and their systematic expression in writing. The inscriptions also show an awareness of and amalgamation with musical systems of other states under the Eastern Zhou world, which emphasized similar ideas. There is no evidence to tie the Zeng musical system to any cosmological ideas; it does not seem to operate according to any perceptions of sound as a cosmological entity. Several elements from this system of music theory remained and are familiar to us from textual sources beginning in the fourth century B.C.E., where their relation to cosmology is evident. Others, like the use of intervals of a Major third and some of the prefixes and suffixes, were lost.

The following sections shows how cosmological ideas were incorporated into the musical system in the fourth and third centuries B.C.E., and how the measurement of tuning standards gained more and more importance, eventually even beyond the musical realm.

Myths: The Origin of Tuning Standards

With the music theory of the Zeng bell and chime sets as one example out of numerous other possible musical systems employed in different states by the mid-fifth century B.C.E., we can argue that the emergence of musical components in texts from the fourth century

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[37] Falkenhausen, *Suspended Music*, 308.

B.C.E. onward signals not a creation, but an incorporation of existing musical ideas. In this section I argue that authors who included emerging cosmological ideas within theories of rulership dug into the world of musical practice, singled out measurable musical components, such as pitches, scales, and intervals, and used them to reflect cosmological order. Thus, ideas regarding tuning, as opposed to discussions about music in general, begin to appear in writings on cosmology.

One way of understanding this is through mythology. Paul R.Goldin has presented and explicated cosmogonic myths in early Chinese texts.[38] Myths on the measuring of sound seem to derive from this category. Myths on the influence of music can involve controlling and transforming the animal world or causing spirits and gods to descend as a sign of moral authority.[39] An example of this type is the myth of Music Master Kui.[40]

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[38] Goldin has debunked the pervasive assertions of some major scholars whose ideas about early Chinese culture as the "Radical Other" to the West necessarily dismissed creation stories appearing in plain sight within the Chinese canon from the excavated versions of the *Loazi* and *Taiyi Shengshui*, to received texts such as the *Huainanzi*, and the *Wuyun Linian Ji*. One example is the position of David L. Hall and Roger T. Ames that "The Chinese tradition, therefore, is "acosmotic" in the sense that it does not depend upon the belief that the totality of things constitutes a single-ordered world." Paul R. Goldin, "The Myth That China Has No Creation Myth," *Monumenta Serica* 56 (2008): 3.

[39] DeWoskin, *A Song for One or Two*, 55. Roel Sterckx, "Transforming the Beasts: Animals and Music in Early China," *T'oung Pao* 86, no. 1/3 (January 1, 2000): 1–46, 3, 27.

[40] The myth of Music Master Kui points to an inherent harmony between the natural, animal, human, and sagely worlds in musical performance. The *Lüshi Chunqiu* 呂氏春秋 contains two versions of this tale. In the first, Kui imitates sounds from nature on his drums, and the chimes of the deity Shang Di on his chimes, see further discussion in Brindley 2012, 18, 28. In its second appearance, Kui is said to correct (*zheng* 正) the six tuning standards and harmonize (*he* 和) them with the five-notes. Music Master Kui appears several times throughout early texts as the codifier of musical performance, and his myth wears different forms. The repeating idea is that of uniting the world of the gods with the world of man through sounds of nature. The cateogires of cosmic and natural tend to mix in the *Lüshi Chunqiu*. I expect to deal with the incarnations of myths on music elsewhere. For a discussion on the syncretic nature of the *Lüshi Chunqiu* chapters on music see Scott Cook, "The 'Lüshi Chunqiu' and the Resolution of Philosophical Dissonance." For a translation of the *Lüshi Chunqiu* versions of the myth see John Knoblock and Jeffrey Riegel, trans., *The Annals of Lü Buwei = [Lü Shi Chun Qiu]: A Complete Translation and Study* (Stanford, Calif: Stanford University Press, 2000), 149, 583.

Two myths on the creation of music tell of the origins of pitches, and thus the construction of a musical temperament. Both relate tuning to cosmology, and in both a principal figure synchronizes a pre-existing cosmic harmony with the world of man by determining a correct set of pitches. Rather than stressing musical performance, or musical composition, these texts focus on the independent importance of tuning. They suggest that since tuning is the backbone of music, it is part of the fundamental connection between man and the cosmos. The earliest myth about tuning appears in the chapter "Five Phases" ("Wuxing" 五行) in the *Guanzi*. The author claims that the purpose of constructing the five-note scale and tuning five bells according to it was part of Huang Di's establishment of government in accordance with Heaven and Earth:

> 昔黃帝以其緩急，作五聲，以政五鍾。令其五鍾，一曰青鍾，大音，二曰赤鍾，重心，三曰黃鍾，灑光，四曰景鍾，昧其明，五曰黑鍾，隱其常。五聲既調，然後作立五行，以正天時。五官以正人位，人與天調，然後天地之美生。
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> In ancient times, by using the slowness and rapidity [of things],[41] Huang Di created[42] five notes to govern five bells.[43] When naming these five

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[41] Rickett translates this section as "blowing softly or strongly," but there is no mention of the verb "blowing" (吹), which would allude to the use of pitch pipes and is used in other chapters of the *Guanzi*. *Yi qi huan ji* 以其緩急 translates as "according to their velocity." This could refer to winds, but in the essay section of the "You guan" chapter it suggests that all affairs have velocities: *ji huan ji zhi shi* 計緩急之事 (Rickett dates the "You guan" to the mid-third century, and translates this sentence in its political context as: "Estimate the relative urgency of affairs"). Thus, I suggest that there is no reason to add a reference to winds, pitch pipes, or *qi* here. W. Allyn Rickett, trans., *Guanzi: Political, Economic, and Philosophical Essays from Early China: A Study and Translation*, vol. 1, Princeton Library of Asian Translations (Princeton, N.J: Princeton University Press, 1985), 192.

[42] The author here uses the verb *zuo* 作 in the sense of "making arise," indicating creation that arises from a harmony that already exists in the world. For additional reading on *zuo* as "making arise" see Michael J. Puett, *The Ambivalence of Creation: Debates Concerning Innovation and Artifice in Early China* (Stanford, Calif: Stanford University Press, 2001), 39–91.

[43] Although the five pitches here are not delegated with the names of the five-note scale (*gong, shang*, *jue*, *zhi*, *yu*), they are mentioned in the "Di Yuan" and "Kui Du" chapters, and so *wu sheng* 五聲 likely refers to their five positions.

> bells, the first was called "Green Bell: Grand Tone" (*dayin* 大音).[44] The second was called "Red Bell: Producing Solemn Thought" (*zhongxin* 重心). The third was called Yellow Bell: Dispersing Light" (*saguang* 灑光).[45] The fourth was called White Bell: Blinding in its brilliance" (*mei qi ming* 昧其名). The fifth was called "Black Bell: Quieting in its Constancy" (*yin qi chang* 隱其常). When the five notes were harmonized, he then established the five phases in order to regulate the seasons of Heaven, and the five bureaus in order to regulate the positions of men. Once Man and Heaven were harmonized, the best of Heaven and Earth was then produced.[46]

This myth is part of a larger discussion in this chapter on the systematization of *yin*, *yang*, *qi*, the seasons, the calendar, and the five phases. Setting specific pitch standards is one of the initial actions taken in the synchronization of the world of man with the cosmos. In this chapter, the author argues for a method other than divination using yarrow or tortoise scapulae, and promotes the numerological use of hexagram lines in the interpretation and detection of cosmic seasonal progression, with its markers at the summer and winter solstices. Then, he employs the myth of Huang Di's adherence to cosmic regulation for this purpose.[47] According to the myth, the regulation of pitches preceded the creation of the five phases. It signified an initial expression of attunement (*diao* 調) between Heaven and Earth,

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[44] "Grand Sound" (*Da Yin* 大音) possibly refers to a sentence in the *Laozi* that describes the Way through the use of antipodes: "Its tone is grand, but is rarely sounded 大音希聲." I say this not only because of the cosmological ideas in the *Guanzi*, but because this sentence is quoted in chapters from later pre-Qin texts that involve sound, such as the *Hanfeizi*, and that involve music, such as the *Lüshi Chunqiu*, suggesting that the idea of an impressive sound, although heard rarely, has great magnitude, was an impactful one. This also fits the characteristic of a grand bell, which is not used daily, but its sound carries meaning and grandeur. In the "Kui Du" chapters, the five notes correlate with the five colors, in the same way as they are here (although not in the same order).

[45] In the "Kui Du" chapter, *Huangzhong* is established as *gong*. Thus, the order of bell names here bears no importance on their musical positions.

[46] Translation closely follows W. Allyn Rickett, trans., *Guanzi: Political, Economic, and Philosophical Essays from Early China: A Study and Translation*, vol. 2, Princeton Library of Asian Translations (Princeton, N.J: Princeton University Press, 1985), 124.

[47] Rickett, 2:121, n.16. Amending *liu duo* 六多 to *liu yao* 六爻. Rickett, 2:123.

in a tangible way, through the sounds of bells. Only once this initial harmony is established (*ran hou* 然後), then the five phases regulate the seasons, and finally, the governmental bureaus fix the positions of man.

The inclusion of this myth, however, is not without some problems. It tells only of the creation of five bells, with five distinct pitches, but the chapter opens by praising the establishment of the five-note scale (*wu sheng* 五聲) and the six tuning standards (*liu lü* 六律), which together create the familiar musical system of early China.[48] Later, the reader is told to "Examine the concordance of the five-note scale, and put it into practice using the twelve bells." It tells the ruler to "Use these to standardize the emotions of the populace" (*shen he qi sheng, xiu shier zhong yi lü ren qing* 審合其聲，修十二鍾。以律人情…).[49] Of the mythical five bells mentioned, only *Huangzhong* is familiar as the name of a tuning standard. The author gives it no particular primacy, and suggests a correlative origin for its name, as part of a set in which each bell is associated with one of the five colors.[50] While scholars are

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[48] The six tuning standards often refer to the entire gamut of twelve pitches, where six *yang* pitches (*lü* 律) are considered principle, and the additional six *yin* pitches are considered secondary (*lü* 呂). Even if the five bells only refer to the five notes, and one may construct them on each of the six or twelve tuning standards, the bell names and their order are still perplexing.

[49] I disagree with Rickett's translation for *"shen he qi sheng, xiu shier zhong yi lü ren qing"* 審合其聲，修十二鍾以律人情: "Pay attention to placing yourself in harmony with the five notes and cultivate the twelve bells so that they may become the pitch pipes for people's emotions." (Rickett, *Guanzi*, 1985, 2:122.). The "Di Yuan" chapter of the *Guanzi* provides the first received textual account of the *sanfen sunyi* 三分損益 formulae for the generation of five tuning standards based on the five-note scale, with *Huangzhong* recognized as the primary position of *gong*. But there has been no definite conclusion for the dating of this chapter. Several authors, including Wang Da, You Yu, Needham, Choy Hun Su, and Rickett agree that it might be a product of the Qin or Western Han, rather than the Warring States. Their conclusions are not based on any of the music theory information in the chapter. The information appearing later in this chapter shows that the *sanfen sunyi* system was well known by the late Warring States. For this reason, I do not contrast the musical system shown here with the myth of Huang Di. See Rickett 1985b, 2:258–59

[50] A different Jing bell 景鍾 is mentioned in the "Discourses of Jin" (*Jin Yu* 晉語) chapter in the *Discourses of the States* (*Guoyu* 國語). In it, the bell is inscribed with a tale that praises Wei Ke's capture of the Qin general Du Hui 杜回, thanks to Wei Ke's relative, who caused Du Hui to fall and be captured by

still uncertain of the origin of *Huangzhong*, we do have evidence of its existence in the pitch names inscribed on the Marquess Yi of Zeng bell set, dated to the mid fifth century B.C.E.

The second myth is that of Ling Lun and the creation of the twelve tuning measures, which also runs along similar lines:

> 昔皇帝使伶倫自大夏之西、崑崙之陰，取竹於嶰谷生，其竅厚均者，斷兩節而吹之，以為黃鐘之管，制十二筒，以聽鳳之鳴，其雄鳴以為六，雌鳴亦為六。天地之風氣正而十二律定，五聲於是乎生，八音於是乎出。[51]
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> In the past, the Yellow Emperor sent Ling Lun 伶倫 to the western Xia [borders], in the shadow of Mount Kunlun, to choose from the bamboo that grows in the [northern] Jie valley. For the even [top and bottom] apertures, [he] punctured two holes and blew into it, thus creating the pipe for the *Huangzhong* pitch and systematized twelve [pitch pipes]. By listening to the cries of the phoenix, he created six [tones] from the male phoenix cries, and from the female cries he made six more. The winds and *qi* of Heaven and Earth organized [them] and the twelve tuning standards were fixed. The five notes were created from this! The eight timbre [categories] emerged from this!

Versions of this myth appear several times between late the Warring States and Eastern Han in texts such as the *Lüshi Chunqiu*, the "Xiuwen" chapter of the *Shuoyuan* 說苑, and *Fengsu Tongyi* 風俗通義. In the *Fengsu Tongyi* version translated above, Ying Shao 應劭 (d. ca. 204 C.E.) gives primacy to the twelve tuning standards over the five-note scale and the

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the Jin, using knots in the grass. Xu Yuangao 徐元誥, *Guoyu Jijie* 國語集解 (Beijing: Zhonghua Shuju, 2002), 404–5.

There is also a repeated anecdote about Duke Jing of Jin (*Jin Jing Gong* 晉景公 r. 599-581 B.C.E.), who built a platform and wished to also cast a great bell (or in one case he had cast a bell and wished to hang it), but the bell is never said to bear his name. The anecdote is a cautionary tale against his extravagance and has no relation to the bell mentioned here. It appears in several texts such as the *Springs and Autumns of Master Yan* (*Yanzi Chunqiu* 晏子春秋), the *Shuoyuan*, and the *Shiji*.

[51] Ying Shao 應劭, *Fengsu tongyi jiaozhu* 風俗通義校注 (Tai bei shi 台北: Mingwen 明文, 1988), 273.

eight timbre categories. Although the myth involves a sage thearch, the Yellow Emperor appears only as a director. He sends Ling Lun to the remote mount Kunlun to regulate and transmit the twelve tuning standards to the human world. Not only are the tuning standards pre-existing in cosmic nature, they were already divided between *yang* and *yin* in the cries of the male and female Phoenix. Ling Lun is presented not as an inventor, but a transmitter, a unifier of *yin* and *yang*, and a systematizer. As DeWoskin has noted: "The fact the twelve pitches were inherent in nature was important. The sage did not create them; he discovered them. Then he found a system by which to make them explicit and useful to mankind."[52] Unlike the *Guanzi* myth, the Ling Lun myth recognizes *Huangzhong* as the foundational pitch within a series of twelve. Moreover, it points to pitch pipes rather than to bells as the original standardizers for the setting of musical tones, and incorporates a human dimension to their discovery. Ling Lun reaches a bamboo field, chooses a bamboo reed, punctures an upper and lower aperture, and blows through it. Mount Kunlun and the mythical phoenix provide the cosmological dimension for this tale.

The earliest available version of this myth, appearing in the *Lüshi Chunqiu,* weaves an additional anecdote to suggest that the casting of bells was done in accordance with the fixed tones of the pitch pipes. It also suggests that this was the origin of the composition of the venerated Ying 英 and the Shao 韶 ritual music used by the Zhou dynasty, thus arguing for an inherent cosmic synchronicity between ancient music, the cosmos, and the calendar:

> 黃帝又命伶倫與榮將鑄十二鐘，以和五音，以施英韶，以仲春之月，乙卯之日，日在奎，始奏之，命之曰咸池。

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[52] DeWoskin, *A Song for One or Two*, 60. Sterckx also emphasized this idea: "The skillful musician or sage holds the task of discovering and observing the patterning of sound and movement in nature and making it explicit to mankind."Sterckx, "Transforming the Beasts: Animals and Music in Early China," 7.

> The Yellow Emperor, once more ordered Ling Lun that together with Rong Jiang[53] he cast twelve bells, by which to harmonize the five tones, and grant the music of the Ying and Shao. On the second month of spring, on *yinmao* day, when the sun reached the Crotch (*kui* 奎) constellation they were to begin playing it. He ordered it to be called the Salt Ponds 咸池.[54]

These myths, which begin to appear around the fourth and third century B.C.E., show that authors utilized tales on the setting and measuring of pitches to argue for the primordial origin of the cosmological philosophy they were promoting. The components of a musical system had long been in practice by then, but now had to be chosen to promote the agenda of investigating the world and the way it works. Tuning, whether of bells, pipes, or strings, were thus related to regulation by adherence to natural cosmic forces. In the Han, with the development and systematization of acoustical and cosmological thought, the dynamics will shift yet again, and the will to keep acoustic principles within adherence to cosmological philosophy will be the impromptu for developments in acoustic theory.

Between Tuning and Cosmology: The The Grand Minister of Music (*Da Siyue* 大司樂) section of the *Rites of Zhou*

The *Rites of Zhou*, known in the Han as the *Officers of Zhou* (*Zhou guan* 周官), is framed as a manual of government offices under the Western Zhou, although there is no agreement on

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[53] A figure by the name of Rong Jiang is not mentioned in other sources.

[54] The Salt Ponds 咸池 is one of the dances in the Zhou dynasty, mentioned in the *Rites of Zhou* as one of the six ritual dances. The Salt Ponds (associated with the sage Emperor Yao, or with the Yellow Emperor) was performed during the annual sacrifice to the earth. See p. 32.

its date of composition and there is a general consensus that it is not historically accurate.[55] Martin Kern deems it "the perfect textual reflection of the bureaucratic idea itself."[56] He sees the *Rites of Zhou* as an "idealized image of perfect bureaucratic order,"[57] composed in the late Warring States or early imperial period, but that, nevertheless, it "reveals a profound knowledge of far older – indeed, Western Zhou – administrative units and their titles that by Warring States times had long been discontinued and replaced."[58] The Grand Minister of Music (*Da siyue* 大司樂) section in the "Spring Offices, [Domain of the] Patriarch of Ancestral Air" ("Chun guan zong bo" 春官宗伯), offers some detailed information about how tuning standards were to be used in ritual performance.[59] However, even if the titles of music administrators date back to the Western Zhou, the details of their responsibilities may have been written at a time when correlative thought was already somewhat advanced, and the author(s) may have wanted to create a simplified version of a late Warring States cosmological theory of music and assign it to the Zhou. Furthermore, as Falkenhausen describes it, this section is "a somewhat garbled locus" for detailed musical information.[60]

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[55] Kalinowski treats the "Spring Offices, [domain of the] patriarch of ancestral air" (*Chun Guan Zong Bo* 春官宗伯) chapter as a mid-third century B.C.E. text. William G. Boltz dates the chapter only as a pre-Han text, probably from no later than the second century B.C.E. See William G. Boltz, "Chou Li 周禮." Kern dates the entire *Rites of Zhou* to the late Warring States or Early imperial periods.

[56] Martin Kern, "Offices of Writing and Reading in the Rituals of the Zhou," in *Statecraft and Classical Learning: The Rituals of Zhou in East Asian History*, ed. Benjamin A. Elman and Martin Kern (Leiden; Boston: Brill, 2010), 65.

[57] Martin Kern, 68.

[58] Martin Kern, 68. Kern bases his conclusion on correspondences between administrative titles in the *Rites of Zhou* and Western Zhou Bronze inscriptions (n.13).

[59] Also translated as Musician-in-Chief. The *Da Siyue* headed the Music Office (*Yue Guan* 樂管), and oversaw the education of state youth for virtues (*de* 德) through ritual musical performance and the main ritual dances was part of his profession, in addition to keeping performance instruments tuned (*yun* 均) according to the tuning measures given to him by the Music Master (*Yue shi* 樂師).

[60] Falkenhausen 1993, 299

Nevertheless, it shows a focus on tuning as an independent and significant part of ritual musical performance, as well as an initial link between sound, *yin yang*, and the winter and summer solstices – a link that would dominate the conceptualization of sound throughout the late Warring States, and the Western Han dynasties.

The sections on the Grand Minister of Music begin by explaining his didactic role in teaching ritual musical performance. The duties included training youth (possibly young blind musicians) by introducing them to military tunes, songs, dances, and rituals. The performance is divided into six sections, each representing or commemorating a different principality and played in sequence. It is probable that each of the tuning standards here stands for an entire scale and not just a single pitch, since songs and melodies are composed with more than just a single note. Another indication is in the next section, which names four pitches for three musical keys, to be used in ritual performance. These sections are free of any correlation to *yin* and *yang*. They give an overview of the purpose of the office, the values it instills, and the musical pieces and accompanying dances it performs:

> 大司樂：掌成均之法，以治建國之學政，而合國之子弟焉。凡有道者、有德者，使教焉；死則以為樂祖，祭於瞽宗。
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> The office of the Grand Minister of Music is in charge of methods for the "Accomplished Standardization."[61] He sets in order the administration of studies for the state foundation, unifying the young generation of the

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[61] *Cheng jun* 成均, "Accomplishing Standards" is a formal name for a particular type of high studies (*da xue* 大學). Zheng Xue cites Dong Zhong Shu's comment that the 成均 refer to the studies of the five emperors (五帝之學). The Eastern Han commentator and Grand Minister of Agriculture (*da sinong* 大司農) Zheng Zhong 鄭眾 (?-87 C.E.), argued that *jun* 均 refers to the standardization between the six tuning standards (*liu lü* 六律) and the five-note scale (*wu sheng* 五聲). The Music Master (*Yue Shi* 樂師), according to him, oversaw the task of setting the correct pitches, and the Grand Minister of Music received the accomplished product – a tuned set of scales – and oversaw its application in musical performance (樂官主調其音，大司樂主受此成事已調之樂). Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, *Zhouli Zhengyi* 周禮正義, 7:1711–13.

state.[62] Generally, those who possess the Way[63] and those whose conduct is virtuous send [their children] to study there. Regarding the deceased, we use them[64] to delight the ancestors, and to worship at the Veneration of the Blind [Musicians].[65]

以樂德教國子：中、和、祇、庸、孝、友。以樂語教國子：興、道、諷、誦、言、語。以樂舞教國子舞《雲門》、《大卷》、《大咸》、《大韶》、《大夏》、《大濩》、《大武》。以六律、六同、五聲、八音、六舞大合樂，以致鬼神示，以和邦國，以諧萬民，以安賓客，以說遠人，以作動物。

By the virtues of musical performance, we instruct the younger generation of the state on loyalty and harmony,[66] veneration and constancy, filial piety and brotherly love. By the lyrics of musical performance, we teach the younger generation of the state aesthetics (?) and guidance, intonation of the Odes, utterance and expression. By the dances of musical performance, we instruct the younger generation of the state to perform Cloud Gate, Great Gathering,[67] Great Salt,[68] Great Shao, Great Xia, Great Hu, and Great Wu. With the six tuning standards, the six corresponding ones, the five-note scale, and the eight timbres, the six dances are greatly unified into a musical performance, through which we reach the spirits, deities, and ancestors, we balance the state, harmonize

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[62] The children of high ranking officials.

[63] Referring to the talented and skilled.

[64] Referring to the youth that participates in the musical performance and sacrifices to deities and ancestors.

[65] The *Gu zong* 瞽宗 could refer to a formal name of an educational establishment, supposedly dating to the Shang dynasty. It may also refer to the *Shen Gu* 神瞽, a different name for an Office of Music (*Yue guan* 樂官), or to divine blind musicians mentioned in "Discourses of Zhou" chapter in the *Discourses of the State* (see translation on p.128). There is no contest that *gu* 瞽, meaning blind, refers to the blind musicians working at the *Gu zong*. It is unclear whether this place was an ancestral temple that employed and educated blind musicians and where sacrifices (*ji* 祭) were made, or, as Wang Shi's 王氏 commentary suggests, a music bureau to which blind youth were sent and where they were trained. Wang glosses *zong* 宗 not as "ancestral temple," but as *zun* 尊, meaning "to venerate," which would mean the name of this institution becomes something like "Veneration of the Blind" and so *ji* 祭 would possibly be translated as "to worship." Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, *Zhouli Zhengyi* 周禮正義, 7:1722.

[66] *Zhong* 中 is glossed as *zhong* 忠. Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, 7:1724.

[67] Following Gu Gongyan's 賈公彥 commentary on *juan* 卷. This list contains seven dances. The first two are associated by legend with Huang Di. The Great Gathering does not appear again, and in the rest of the verses only six dances are included.

[68] Referring to the Salt Ponds 咸池, mentioned again below.

the people, host our guests, persuade others from afar, and set all things in motion.

The following section begins to lay out the organization of ritual music and dances according to three main sacrifices: sacrifices to heavenly deities, sacrifices to earthly beings, and ancestral sacrifices. The musical content in the two following sections is not consistent, probably because these are two separate rituals. The first section discusses a performance cycle that is to be conducted six times. It includes the vocal songs, musical pieces, and dances of six ruling periods, from mythological times [Huang Di] to the present Zhou, and their associated sacrifices:

乃分樂而序之，以祭，以享，以祀。乃奏黃鐘，歌大呂，舞《雲門》，以祀天神。乃奏大蔟，歌應鐘，舞《咸池》，以祭地示。乃奏姑洗，歌南呂，舞《大韶》，以祀四望。乃奏蕤賓，歌函鐘，舞《大夏》，以祭山川。乃奏夷則，歌小呂，舞《大濩》，以享先妣。乃奏無射，歌夾鐘，舞《大武》，以享先祖。凡六樂者，文之以五聲，播之以八音。凡六樂者，一變而致羽物及川澤之示，再變而致羸物及山林之示，三變而致鱗物及丘陵之示，四變而致毛物及墳衍之示，五變而致介物及土示，六變而致象物及天神。[69]

Then, they divide the musical performance and organize it according to the *Ji*, *Xiang*, and *Si* sacrifices. They play in the *Huangzhong* [key], sing in the *Dalü* [key], and dance the Cloud Gate, while performing the sacrifice for heavenly deities. They play in the *Dacou* [key], sing in the *Yingzhong* [key], and dance the Salt Ponds, while performing the earth sacrifices. They play in the *Guxi* [key], sing in the *Nanlü* [key], and dance the Great Shao, while performing sacrifices to the four directions. They play in the *Ruibin* [key], sing in the *Hanzhong* [key], and dance the Great Xia, while performing sacrifice the mountains and rivers. They play in the *Yinze* [key], sing in the *Xiaolü* [key], and dance the Great Hu, while performing sacrifices to the ancestral mothers [of the Zhou]. They play in the *Wuyi* [key], sing in the *Jiazhong* [key], and dance the Great Wu, while performing sacrifice to the ancestral fathers [of the Zhou].

[69] Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, *Zhouli Zhengyi* 周禮正義, 7:1739–56.

> These six musical performances are patterned on the five-note scale, and spread across the eight timbres. These six musical performances, the first performance cycle, reaches the feathered creatures and the marshes. The second cycle reaches creatures without hides and the mountains and forests. The third cycle reaches scaled creatures and mounded hills. The fourth cycle reaches feathered creatures and low leveled planes. The fifth cycle reaches shelled creatures and the earth. The sixth cycle reaches all animals and deities.

Later in the "Chunguan" chapter, we are told that the Grand Music Master (*Da Shi* 大師) oversees harmonizing the *yin* and the *yang* of the six tuning standards (*lü* 律) and the six corresponding standards (*tong* 同), and it provides the following division of pitches:

Yang pitches (*yang* 陽):	Corresponding pitches (*tong* 同):
Huangzhong 黃鐘、*Dacou* 大蔟、*Guxi* 姑洗、Ruibin 蕤賓、*Yize* 夷則、*Wuyi* 無射	*Dalü* 大呂、*Yingzhong* 應鐘、*Nanlü* 南呂、*Hanzhong* 函鐘、*Xiaolü* 小呂、*Jiazhong* 夾鐘

He patterns these pitches according to the five-note scale, and apportions it throughout the eight timbres. He uses the six *yang* tuning standards to teach the Six Virtues (*liu de* 六德) and the *yin* tuning standards to listen to the sounds of the army, and [thereby] to determine good and bad omens. When matching this division with tunes and songs in the above section, it becomes clear that performers did not arbitrarily choose a pitch to begin playing or singing. The starting point, the chosen scale, had an additional meaning – it reflected a harmonization between *yin* and *yang*. For each tune played on a *yang* scale, a song was sung in its corresponding *yin* scale, in order.[70]

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[70] In his 1851 translation, Biot hypothesized that the difference between the instrumental (*zou* 奏) and the sung (*ge* 歌) lies in the ability of instruments to play accurately, and that this task was more difficult for the singing voice. This may be true for impromptu on-demand playing of single pitches or singing particularly difficult intervals, but it is does not necessarily apply to singing melodies. In this case, it is apparent that the difference lies in the correspondence between *yang* and *yin* based modes, rather than accurate executional

The following table presents the organization of these six musical performance and clarifies the link between musical pieces, played in *yang* keys, and vocal songs, played in their corresponding *yin* keys:

The Music of the Six Rulers				
Tune in Yang key	**Song in Yin key**	**Regional Dance**	**Ritual**	**Ruler**
Huangzhong 黃鐘	*Dalü* 大呂	Cloud Gate 雲門	Heavenly Deities 祀天神	Huang Di
Dacou 大蔟	*Yingzhong* 應鐘	Salt Ponds 咸池[71]	Earthly Beings 祭地示	Huang Di, Yao
Guxi 姑洗	*Nanlü* 南呂	Great Shao 大韶	Four Directions 祀四望	Shun
Ruibin 蕤賓	*Hanzhong* 函鐘[72]	Great Xia 大夏	Mountains and Rivers 祭山川	Xia (ruler Yu 禹)
Yize 夷則	*Xiaolü* 小呂[73]	Great Hu 大濩[74]	Ancestral Mothers 享先妣	Shang (King Tang)
Wuyi 無射	*Jiazhong* 夾鐘	Great Wu 大武	Ancestral Fathers 享先祖	Zhou (King Wu)

According to the "Addition and Subtraction by a Third" (*sanfen sunyi* 三分損益) formula for generation of tuning standards, which was fully in use by the mid-third century

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abilities. Jean-Baptiste Biot and Stanislas Julien, eds., *Le Tcheou-Li: Ou, Rites Des Tcheou. 1851. Reprint.*, trans. Edouard Biot, 2nd ed., vol. 2, 3 v. (Taipei: Ch'eng Wen, 1975), 33 n.3.

[71] Zheng Xuan's commentary attribues the Salt Ponds musical performance to Huang Di, with additions and standardizations added by Emperor Yao. (黃帝所作樂名也，堯增脩而用之。)

[72] Linzhong 林鐘

[73] Zhonglü 中呂

[74] Read as *hu* 護. According to Zheng Xuan's commentary, it was thought to be the music of Tang, the last ruler of the Shang dynasty (*Shang Tang* 商湯)

B.C.E., each *yang* tuning standard produces a *yin* one. In the above division, instrumental tunes and vocal songs are not paired so that a *yang* tuning standard corresponds to the *yin* tuning standard it generates. Rather, they seem to be treated as two separate sets, in which each of the *yang* tuning standards is a tone apart from each other, and each of the *yin* songs is also a tone apart from each other.[75]

This predetermined interval of one tone between each of the *yang* tunes creates a set of six performances in ascending pitch order. If we arbitrarily assign the western note *do* to *Huangzhong,* the pitch order for the *yang* tunes would be: do, re, me, fa#, sol#, la#. The *yin* songs in this performance would be sung in a series of descending pitches. If *Huangzhong* is set as do, then *Dalü* is do#. The pitch order of the *yin* songs then becomes: Do#, si, la, sol, fa, mi*b*. When ascending and descending are combined, a chromatic twelve pitch base scale results.[76]

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[75] Mathematically, according to the *sanfen sunyi* method, the ratio between these tones is equivalent to a Pythagorean whole tone, which has a ratio of 9/8 (or an interval of 203.91 cents). However, it is important to note that we do not know how these ratios were translated from theory into practice and whether musicians had a way of maintaining such accuracy. They certainly were not able to do so in fixed-tune instruments such as bells and chimes, but they may have been able to generate these intervals on strings, depending on the tuning method they used. If they simply tuned by ear, according to fifths, the result would also be a Pythagorean scale. This does not mean that accurate tuning was not important or implemented. We simply do not know how or whether it was implemented correctly, or what they considered to be the correct matching between ratios and played pitches. Suffice to say that as far as performers and those in charge of tuning were concerned, they assumed they adhered to these tuning standard, and made every effort to do so.

[76] Commentators enjoyed theorizing on this section, providing several options for scale generations. They mainly rely on anachronistic sources like the *HHS, Jin shu*, and others, I am reluctant to provide them here. The 19th century commentator Chen Li 陳豐 does offer a reasonable explanation for the statement *"jie wen zhi yi wu sheng*: *gong, shang, jue, zhi, yu"* 皆文之以五聲：宮、商、角、徵、羽, arguing that each of the tuning standards had five modulations. For example: a five tone scale would be constructed on *Huangzhong* at the first position (*gong* 宮), and then begin to circulate with *Huangzhong* in the second position, and then in the third, fourth and fifth. When applied to all twelve tuning standards, the result is sixty options for modulations. This is important, if only to strengthen the argument that early Chinese texts discussed entire scales and not simply single pitches — despite not saying so directly — and to show the rich possibilities, at least in theory, of the musical world of early China. Twelve tuning standards and five positions are only the basis from which various scales could be constructed. See Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, *Zhouli Zhengyi* 周禮正義, 7:1833.

The above section on the music of the six rulers allows us to understand the interaction between the rising *yin* and falling *yang* and the way the sources say musicians applied them in the performance of ritual.

The following section discusses three rituals: a ritual to heavenly deities, conducted on the summer solstice; a ritual to heavenly beings, conducted on the winter solstice; and a ritual to the ancestors.[77] The choice of scales for each event is, again, important. The musicians must play the correct scale each time, and the pitches and positions of each scale are also detailed, although the way they use them is unclear:[78]

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[77] The paragraph concerning ancestral sacrifices is worded differently the those on the winter and summer solstices. It includes vocal songs and musical pieces, and they are not associated with a geographic location, except the ancestral shrine.

[78] The scales in this section are unfamiliar. When comparing them to the *sanfen sunyi* system of pitch generation with *Huangzhong* set as the first pitch, they neither produce nor follow each other. The *Huangzhong* scale, if rearranged, has the option of creating the chromatic tetrachord equivalent to: si, do, do#, re. Either the pitch names do not adhere to the rules of the *sanfen sunyi* system, in which case there is no telling the sonority of these scales, or they were chosen according to another aesthetic standard for scale

凡樂，圜鐘[79]為宮，黃鐘為角，大蔟為徵，姑洗為羽，雷鼓雷鼗，孤竹之管，雲和之琴瑟，《雲門》之舞；冬日至，於地上之圜丘奏之，若樂六變，則天神皆降，可得而禮矣。

In musical performance, *Huanzhong* sits in the *gong* position[80], *Huangzhong* in *jue*[81], *Dacou* in *zhi*[82], *Guxi* in *yu*.[83] The Thunder Drum[84] and the Thunder Hand-drum, the rootless bamboo pipes, the *qin* and *se* of Yunhe mountain (Harmonious Cloud), and the Cloud Gate dance, on the winter solstice, are played on the ground of Huan Qu. After six musical performance cycles the heavenly deities descend and they can access and perform ritual to them!

凡樂，函鐘[85]為宮，大蔟為角，姑洗為徵，南呂為羽，靈鼓靈鼗，孫竹之管，空桑之琴瑟，《咸池》之舞；夏日至，於澤中之方丘奏之，若樂八變，則地示皆出，可得而禮矣。

In musical performances[86], Hanzhong sits in the *gong* position. Dacou in *jue*. Guxi in *zhi*. Nanlü in *yu*. The Spirit Drum, and Spirit Hand-drum, the rootless bamboo pipes, the *qin* and *se* of Kongsan mountain, and the Salt Pond dance, on the summer solstice, are played at the marshes on Fang Qu altar. After eight musical performance cycles the earthly beings come out, and they can access and perform ritual to them!

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production. The positions of the five-note scale seem to be the same as they appear in other texts, except that these scales drop the second position *shang* 商, and use only four pitches per scale.

夾鍾 re#, 黃鐘 do, 大湊 re, 姑洗 mi; 林鐘 sol, 大湊 re, 姑洗 mi, 南呂 la; 黃鐘 do, 大呂 do#, 大湊 re, 應鍾 si. (if positions 1= 宮= do; 2= 商= re; 3= 角= mi; 4 = 徵= sol; 5 =羽 =la).

[79] *Huanzhong* 圜鐘 is a different name for the *jiazhong* 夾鍾 pitch. Sun Yirang 孫詒讓, Wang Wenjin 王文錦, and Chen Yuxia 陳玉霞, *Zhouli Zhengyi* 周禮正義, 7:1757.

[80] First position. Equivalent to do.

[81] Third position. Equivalent to mi.

[82] Fourth position. Equivalent to sol.

[83] Fifth position. Equivalent to la. The text does not mention the second position of the scale (*shang* 商).

[84] The commentaries disagree on whether the Thunder Drum was an eight-sided, six-sided, four-sided, or two-sided drum. This drum is also mentioned under the musician personnel working in the Earth Office (*Di guan* 地官). The size of this drum is unknown. Tang dynasty's Yang Liang's commentary to the *Xunzi* "解蔽" chapter describes it as a large drum, but he seems to infer this only based on the analogy of its sound to that of thunder (*leigu, da gu*, *sheng ru lei zhe* 雷鼓，大鼓，聲如雷者). The shape of the Spirit Drum (*lei gu* 靈鼓) in the following section is also undetermined.

[85] *Hanzhong* 函鐘 is a different name for the *linzhong* 林鐘 pitch. 1757.

[86] This probably refers to the music performed in these three types of sacrifice:

凡樂，黃鐘為宮，大呂為角，大蔟為徵，應鐘為羽，路鼓路鼗，陰竹之管，龍門之琴瑟，《九德》之歌，《九韶》之舞；於宗廟之中奏之，若樂九變，則人鬼可得而禮矣！

In musical performances, *Huangzhong* sits in the *gong* position. *Dalü* in *jue*. *Dacou* in *zhi*. *Yingzhong* in *yu*. The Path drum and Path hand-drum, bamboo pipes from the north side of the mountains,[87] the *qin* and *se* of Longmen, the Nine Virtues songs, and the Nine Shao dances are played at the royal ancestral shrine. After nine musical performance cycles, they can access and perform ritual to the spirits of the deceased!

The main purpose of teaching and performing ritual, according to this text, was to cultivate moral values, teach essential performance skills, and eventually through music, song, dance, and ritual, cause deities to descend, influence earthly beings, and communicate with ancestors. But if we look at the responsibilities of the Grand Minister of Music and the Grand Music Master in the preparation for musical performances, there is an evident emphasis on tuning in the sense of picking, choosing, and dividing the music according to keys: The tune accompanying the sacrifice for earthly beings must be performed in the key of *Taicou* 大蔟, and the accompanying song must use the corresponding *Yingzhong* 應鐘 key; any other way would be incorrect. Of course, we cannot know the frequency of these pitches, or how the tunes and songs sounded. We cannot know the number of pitches from the scale used in each tune, nor can we know the ways music masters determined whether instruments were in or out of tune. We can, however, recognize the emphasis on the technical concept of tuning.

[87] The refence to bamboo pipes from the northern valley may have some connection to the Ling Lun myth, which first appears later, in *Lüshi Chunqiu*. Ling Lun is sent to a bamboo field located on the north, shaded side of mount Kunlun, where he finds and picks the bamboo that becomes the standard for the *Huangzhong* pitch.

As for cosmology, Lewis argues that in the *Rites of Zhou* "the offices are organized to function as a symbolic reproduction of the structure and workings of the cosmos."[88] He treats the entire text as "a text created as a model of the world."[89] Kern objects to this classification, stating that "While any numerologically driven order, one might argue, is inherently cosmological, I am reluctant to see the text in these terms. It is not how it was discussed during its early reception."[90] While it may not have been originally interpreted as a cosmological text, the sections focusing on musical performance certainly include a cosmological outlook on the function of sound. The section on the music of the six rulers shows not only a direct association between fundamental pitches (and their associated scales) and *yin* and *yang*, but also a sensitivity to their complementarily undulating nature, in which *yang* rises as *yin* falls. The section on the three rituals shows that musical keys were associated also with the major calendric events of the summer and winter solstices. Lewis also argues that the numbers five and six are significant as numerological organizing principles.[91] The five-note and six-tuning-standards musical system in the "Grand Music Minister" section seems to bear out his argument, but it is not the sole evidence for a cosmological approach to sound.

The Grand Music Master (*Da Shi* 大師) is "in charge of the [tuning of the] six tuning standards and the six corresponding ones, harmonizing the sounds of the *yin* and the *yang*." (*zhang liu lü, liu tong, yi he yin yang zhi sheng* 掌六律、六同，以合陰陽之聲). Later, the

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[88] Mark Edward Lewis, *Writing and Authority in Early China*, SUNY Series in Chinese Philosophy and Culture (Albany, NY: State Univ. of New York Press, 1999), 44.

[89] Lewis, 48.

[90] Martin Kern, "Offices of Writing and Reading in the Rituals of the Zhou," 68 n.12.

[91] Lewis, *Writing and Authority in Early China*, 47.

Tone Monitor 典同 is said to "oversee the harmonization of the six-tuning-standards and the six corresponding ones, by distinguishing the sounds of heaven, earth, the four directions, and *yin* and *yang* and applying them to make musical instruments (*zheng liu lü, liu tong zhi he*, *yi bian tian di sifang yin yang zhi sheng*, *yiwei yue qi* 掌六律、六同之和，以辨天地四方陰陽之聲，以為樂器). These show a cosmological conceptualization of the function of sound.

These *Zhouli* sections do not present a detailed numerological system or even very detailed calendrical scheme wrapped around the musical system. Such an emphasis on mathematical accuracy of tuning measures, the investigation of the cosmos through numbers, the manifestation of cosmic forces through sound as a vibration of *qi*, which are apparent in later sources, is not yet present here. As I will show presently in the discussion of the Fangmatan daybooks, the numerological aspect of tuning measures became much more complex and systematized by the mid second century B.C.E. Nevertheless, the division of the twelve tuning standards into a set of rising *yang* and a set of falling *yin,* their application in the state's tunes, songs, and dances, and the subsequent emphasis on specific pitches and keys to be used in solstitial ritual, do show an emerging connection between cosmology, the calendar, and tuning.

Regarding the date of this text, it is significant that the nomenclature of the pitches resembles closely, but not exactly, the Zeng inscriptions.[92] An identical nomenclature does

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[92] Falkenhausen also mentions the ascending and descending characteristics of the *lü* and *tong* pitches in this section of the *Rites of Zhou*. He treats each set as a scale in its own right: "Both enumerations are arranged as if constituting a scale; but while the *lü* are listed in an ascending order of pitch (or rather, of the pitch of their *gong* [do]), the *tong* are listed in descending order. In the section on the Great Director of Music (*Da Siyue* 大司樂), each *lü* on the ascending scale is paired with a *tong* on the descending scale, the interval between the two *gong* thus increasing from pair to pair." I see these pitches as fundamentals on which scales are to be constructed. My view is based on their function in tunes and songs, which, as I have mentioned, used more than a single pitch, and because the text designates the five notes and six-tuning-

not appear in other textual sources. Additionally, the division of the twelve tuning standards into two sets of *yin* and *yang* that do not generate each other is also unique to this text. Lastly, the text indicates *Huanzhong* 圜鐘 at the *gong* position in the ritual held at the winter solstice. It is possible that *Huangzhong* was not yet fixed at the winter solstice as it is in later texts. These three features may indicate an early date for the content, not only the administrative titles.

The Daybooks of Fangmatan: Divining With Tuning Standards

Daybooks (*rishu* 日書) refer to a type of almanac marking auspicious and inauspicious days. Named after two almanacs discovered in 1975 in Shuihudi 睡虎地 (Hubei province) in a tomb of a Qin official named Xi 喜, who died in 217 B.C.E.[93] Local administrators as well as high bureaucratic officials commonly consulted daybooks to determine public as well as private activities. Almanacs were clearly a thriving genre in third century B.C.E. China. They contained correlative schemes, which incorporated the heavenly stems and earthly branches, seasons, times of day, the twenty-eight lodges[94] and their planets, the five-note scale, five

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standards as the base for the musical system. Thus, I think we should not view each set as a scale, but as six options for fundamentals. Falkenhausen does not discuss the application of these principles in ritual performance. Falkenhausen 1993, 293–96

[93] For an analysis of the Shuihudi texts see Marc Kalinowski, "Les Traités de Shuihudi et l'hémérologie Chinoise a La Fin Des Royaumes-Combattants," *T'oung Pao* 72, no. 4/5 (1986): 175–228. These daybooks were in common use, and by 2011, around twenty such texts dating from the Warring States to the Han have been discovered in all. Wang 2006, 87–88

[94] Following Christopher Cullen's translation of *Xiu* 宿 as "lodges." The twenty eight *xiu*" (*ershiba xiu* 二十八宿) are often mistranslated as "the twenty eight lunar mansions" or "lunar lodges," when they did not have anything to do with the position of the moon: "The instances of 宿 *suk/sù, 宿 *sukh/xiù and 舍 *lhah/shè in technical literature of the early imperial age do not indicate that any of these terms have a specifically lunar reference, or are thought by anybody writing in that period to have such a reference. There is therefore no reason to continue using such terms as 'lunar lodges' or 'lunar mansions' as

colors, five phases, five directions, animals, and more.[95] Wang Aihe has argued that the almanacs attest to the spread of correlative cosmology to the middle and lower social tiers, "becoming a common cultural phenomenon that paved the way for the unification of a vastly diverse populace."[96]

In 1986, at Fangmatan 放馬灘, Tianshui, in Gansu province, archaeologists unearthed two additional daybooks, which they dated to the mid-third century B.C.E.[97] These daybooks belonged to a local man named Dan 丹, who possibly carried some importance in the local community, and was thought to possess magical powers.[98] The Fangmatan manuscripts are similar to the Shuihudi ones, and an antecedent to both is the oldest surviving almanac from the Chu Silk Manuscript, unearthed in Changsha (Henan province), and dated to ca. 300 B.C.E.[99] The differences between the Chu Silk Manuscript and the daybooks make apparent the chronological development in systems of correlative cosmology around the third century B.C.E.

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translations for these words, and since such translations inevitably give the reader a false impression that there is reason to claim a lunar connection, they should be discontinued." Christopher Cullen 2011, 93

[95] Donald J. Harper, "Warring States Natural Philosophy and Accult Thought," in *The Cambridge History of Ancient China. From the Origins of Civilization to 221 B.C.*, vol. 1 (Cambridge: Cambridge University Press, 1999), 843; Kalinowski, "Musique et Harmonie Calendaire," 101.

[96] Wang Aihe, *Cosmology and Political Culture in Early China*, 88. For a detailed study on the daily functioning of local government in the Warring States see: Daniel S. Sou, "In the Government's Service : A Study of the Role and Practice of Early China's Officials Based on Excavated Manuscripts" (2013).

[97] For a transcription of the daybooks, which appear on scroll B see *Yan Changgui 晏昌貴, "Tianshui Fangmatan Qin jian Yi zhong rishu fen pian shiwen (gao)* 天水放馬灘秦簡乙種《日書》分篇釋文(稿)," in *Jianbo* 簡帛, vol. 5 (Shanghai: *Shanghai guji chubanshe*, 2010), 17–42. Kalinowski provides "239 B.C.E. or a bit later" as the date for the Fangmatan no.1 tomb, by relying on an inscription of the date on one of the bamboo strips. See Kalinowski, "Musique et Harmonie Calendaire," 103.

[98] Wang Aihe, *Cosmology and Political Culture in Early China*, 88. The name Dan 丹, meaning "cinnabar," too meaningful in this context to be accidental. It may be related to his role and divinatory abilities.

[99] Harper 1999, 845–47

In relation to sound, both the Shuihudi and Fangmatan daybooks include pitch generation formulas and they place note names in the context of correlative cosmology. Information on the numerical representation of tuning is richest in scroll B of the Fangmatan daybooks (hereafter FMTB).[100] It provides a formula for pitch generation, two methods to calculate the twelve tuning standards, associates the twelve standards with geographical locations, and includes numerous divination scenarios and instructions involving the five-note scale and the twelve tuning standards.[101]

Regarding the numerous slips that discuss divination using tuning standards, Kalinowski argues that "it is not possible to determine the original number of texts and their arrangement in sections or to be certain of their place in the daybooks manuscript." Cheng Shaoxuan 程少軒 argues that FMTB is an independent work, which he titles "A Cosmic Board Divination Book on the Chinese Twelve Tones" (*zhonglü shizhan* 鐘律式占), and is separate from the almanac part of the daybooks. On the cosmological aspect of the work, he argues that this Divination Book represents "a transitional piece between early cosmic board divination books and more mature works from the Han dynasty," and relates it to titles in the *Yiwen zhi* 藝文志 bibliographic section in the *Han shu* that concern tuning standards, *yin yang*, and the five phases. Chen divides the Divination Book into three independent, interchangeable sections: cosmic board diagrams (*shitu* 式圖) and explanations, which include a chart of the twelve tuning standards, and the precursor to the Containing Tones divination system (*nayin* 納音. See p. 45); divination interpretations (*guaci* 卦辭 and *zhanci*

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[100] This section relies on Kalinowski's analysis in Kalinowski, "Musique et Harmonie Calendaire."

[101] I have not been able to confirm most of the locations on the list. More work is required in order to determine whether these are actual locations or mythical ones.

占辭), among them the *Auspicious and Inauspicious Prediction of the Twelve Tuning Standards* 十二律吉兇, and *Divination of the Twelve Tuning Standards* 十二律占; and explanations about various divination methods (among them, divination using the numerical values of tuning standards 以鍾律數占卜).[102] Unfortunately, the poor condition of the bamboo slips belonging to the third section does not allow a reliable reconstruction of these explanations.

Overall, FMTB shows that the local population in the late Warring States employed tuning formulae, a cosmologically significant numerical expression of tuning standards and interval ratios, outside the realm of music, as an independent tool for assessing cosmic and geomantic conditions in divination. No instruments are required to use this type of system, which attempted to determine favorable actions and auspicious dates.[103] Still, each number represents a tuning standard, the positions of note names and tuning standards are significant in the divination, as is the choice of tuning standards for particular days and times. It is wise to remember, however, that this was only one out of several uses for sound measurement in early China.

In the discussion on the applications of sound in early China, I argue that some instances point to its use in musical practice, while other instances point to its use in non-musical practices, which do not require audible sound. The measuring of sound played a significant role in ritual musical performance before the unification of China, as the above examples from the *Zhouli* and the *Guanzi* show. In Part Two of this dissertation, I show that Han officials had every intention of using tuning formulae in ritual music (for strings and pipes), especially when it concerned solar events, such as the solstices. Even if we are not yet

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[102] Cheng Shaoxuan 程少軒 2012, 245-6

[103] Kalinowski, "Musique et Harmonie Calendaire," 133.

certain of how theory was put into practice and whether it was accurate, numerous examples take for granted the application of these theories to instruments, in addition to Eastern Han examples that lament the loss of knowledge of how to apply these measurements. I point to FMTB, rather than the treatises, as an example of a theoretical approach to tuning in which numbers were an essential part of the technique to determine future actions. FMTB treats sounds as among the quantifiable elements that make up the world around us, rather than as a musical tool for composition.

According to Kalinowski, FMTB contains a single divination system, but we are still uncertain of the details of its execution.[104] First, it describes the calculation of tuning standards using the "Addition and Subtraction by One Third" pitch generation system (*sanfen sunyi*). Until the excavation of this manuscript, that system was available only in later received sources, of which the earliest is the "Di Yuan" chapter of the *Guanzi*, which has not been definitely dated. The daybooks of Fangmatan are the first datable evidence for the use of mathematical ratios to define intervals and calculate tuning standards in the third century B.C.E. It validates the practical use of interval ratios in the hemerological arts in the late Warring States.

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[104] Donald J. Harper and Marc Kalinowski, eds., *Books of Fate and Popular Culture in Early China: The Daybook Manuscripts of the Warring States, Qin, and Han*, Handbook of Oriental Studies = Handbuch Der Orientalistik. Section 4, China, volume 33 (Leiden ; Boston: Brill, 2017), 473 (Appendix C). Kalinowski has also shown that daybooks hold the antecedents what was known in the third century as the Nayin 納音 (Containing Tones) system, which "determines the note and agent governing a person's fate according to birth year." The first appearance of the Nayin system is in the *Baopuzi* 抱朴子. The daybooks contain the "oldest evidence of the five-notes (*wuyin* 五音) sexagenary system, in which the sixty binoms are arranged in five groups of twelve binoms each and each group is associated with one of the five notes (*gong* –F, *shang*-G, *jue*-E, *zhi*-C, *yu*-D) and five agents." (it is possible that *jue* should have been and A instead of E here). Harper and Kalinowski, 470.

In the *sanfen sunyi* method, the practitioner chooses a number to represent the initial pitch from which calculation begins. This number is then divided by three and the result is either added or subtracted from the initial number. The result is the next pitch. The ratio between the two represents a musical interval. Reducing by one third produces a ratio of 3/2 (between the initial pitch and the following one), which is known as an interval called a fifth. Adding one third produces a ratio of 4/3, known as an interval called a fourth. This cyclical system applies to strings and pipes (not to bells, since their construction is much more complex, and they are not tuned by lengthening and shortening the bell).

The ratios are only a mathematical expression of intervals, and in later texts, such as the *Shiji*, they are accompanied by string and pipe measures as practical means to verify the calculations. The choice of a number to begin the calculations is arbitrary, since these are ratios and not fixed lengths. The chosen number merely needs to be divisible by 3 and to produce an integer. When this tuning formula appeared in early China, the most popular choice for the number of the fundamental was 81, attached to *Huangzhong*. In the discussion of List 2, I elaborate on the significance of the number 81 and the problems that arise when one wants to generate more than five tones.

After introducing the *sanfen sunyi* formula, the FMTB subtracts one third from the numerical value of *Huangzhong* (81-27=54). This step is also known from Western Han textual sources such as the "Celestial Patterns" ("Tianwen" 天文) chapter in the *Huainanzi*, and the "Book on Tuning Standards" ("Lü shu" 律書) in the *Shiji*.[105] This initial generation

[105] The result is apparent in the octaves on which the following tuning standards are produced, due to the general direction of the cycle. When the *Guanzi* scheme produces only the five-note scale, it would create the ascending sol-la-do-re-mi (when arbitrarily starting from do). The FMTB scheme would create do-re-mi-sol-la (when arbitrarily starting from do). When producing the twelve tuning standards, the problem is that the *Guanzi* scheme would not create an ascending continuous chromatic scale. Some of the tuning standards would have to jump an octave in order to create a neatly organized scale. This is not difficult to

differs from the "Di Yuan" chapter, which begins by adding one third to the number assigned to *Huangzhong* (81+27=108).[106] An easy way to understand it is to say that the *Guanzi* is a downwards/upwards cycle, while FMTB is an upwards/downwards cycle.[107]

Second, FMTB contains three separate lists, each containing the generation of twelve tuning standards:

List 1 contains only the names of the tuning standards and the initial process of upward or downward pitch generation, without providing any numerical value to them:

List 1.

> 黃鐘下生林鐘, 林鐘生大簇, 大簇生南呂, 南呂生姑洗, 姑洗生應鐘, 應鐘生蕤賓, 蕤賓生 大呂, 大呂生夷則, 夷則生夾鐘, 夾鐘生毋射, [毋射生中呂].[108]
>
> *Huangzhong* downward generates *Linzhong*. *Linzhong* generates *Dacou*.[109] *Dacou* generates *Nanlü*. *Nanlü* generates *Guxi*. *Guxi* generates *Yingzhong*.

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do, but should be pointed out. Moreover, the *Guanzi* does not offer an option for twelve tuning standards, but rather it stops at five. If we use the FMTB scheme, this problem does not exist.

[106] Chen Yingshi argues that the *Lüshi Chunqiu* "Tones and Tuning Standards" (*yinlü* 音律) chapter also presents the same scheme, although not as clearly, since no numerical values are added in the *yinlü* chapter. The confusion arises from the definition for upwards generation (*wei shang* 为上) and downwards generation (*wei xia* 為). For further discussions on upward and downward pitch generation see Chen Yingshi 陳應時, "*Guanzi, Lüshi chunqiu de shenglü fa ji qita* « 管子 » « 呂氏春秋 » 的生律法及其他," *Huangzhong* (*Wuhan Music Conservatory*)《黃钟(武汉音乐学院学报)》, no. 3 (2000): 64–68. Kalinowski 2011, 213–127; Cheng Shaoxuan 程少軒, "Fangmatan Jian," 260.

[107] Thus, according to the *Guanzi*, if we are to arbitrarily select the note middle do^0, and assign it the numerical value 81, the following note adds a third of 81 to its value, thus 108 is its numerical value. The ratio 81/108 can be reduced to 3/4 – a fourth. The result would be sol^{-1}, an interval of a fourth below middle do. The following note would be re^0, an interval of a fifth above that lower sol^{-1}.

According to FMTB, if we are to arbitrarily select the note middle do^0, and assign it the numerical value 81, the following note reduces a third from 81, thus 54 is its numerical value. The ratio 81/54 can be reduced to 3/2 – a fifth. The result would be sol^0, an interval of a fifth above middle do. The following note would be d^0, an interval of a fourth below that lower sol^{-1}.

[108] These three lists follow the Chinese transcription in Cheng Shaoxuan 程少軒 2012-2. In List 1. Kalinowski inverts the last three slips (which contain the last three pitch generations) to arrive at the correct order of names and numbers. He attributes the mistake to the copyist, and shows how the correct order was known and is verified in the List 2. I concur and follow this argument.

[109] Usually written as *Taicou* 太簇.

Yingzhong generates *Ruibin*. *Ruibin* generates *Dalü*. *Dalü* generates *Yize*. *Yize* generates *Jiazhong*. *Jiazhong* generates *Wuyi*.[110] [*Wuyi* generate *Zhonglü*].

List 2 presents a geomantic correlation between tuning standards and geographical locations. It also assigns a numerical value to each tuning standard starting at 81 for *Huangzhong*. The tuning standards are presented in ascending chromatic order rather than order of generation:

List 2.

黃鐘八十一, 課山; 大呂七十六, □山; 大簇七十 二, 參阿; 夾鐘六十八, 參阿; 姑洗六十四, 陽谷; 中呂六十, 俗山; 蕤賓五十七, 冕都; 林鐘 五十四, 俗山; [夷則五十一, □□]; 南呂卅八, 俗山; 毋射卅五, 昏陽; 應鐘卅三, 並閾.

The information is given concisely in the following table:

	Name	Harmonic Number	Location		Name	Harmonic Number	Location
1	*Huangzhong*	81	Mount Ke	**7**	*Ruibin*	57	City of Mian
2	*Dalü*	76	Mount [?]	**8**	*Linzhong*	54	Mount Su
3	*Dacou*	72	Shen Hill	**9**	*Yinze*	51	—
4	*Jiazhong*	68	Shen Hill	**10**	*Nanlü*	48	Mount Su
5	*Guxi*	64	Yang Valley	**11**	*Yinshi*	45	Couchant Yang
6	*Zhonglü*	60[111]	Mount Su	**12**	*Yingzhong*	**43**	並閾?

[110] Usually written as 無射.

[111] The following tuning standard would close the twelve-pitch cycle on a pitch that is an octave higher from the starting point of *Huangzhong*. The ratios, however, do not close this cycle perfectly (the adding a third to 60 equals 80 instead of 81), but this was not discussed as a problem requiring a solution until the Western Han (see Part Two of this dissertation).

Western scholars have named the assignment of numerical values to tuning standards beginning at 81 "Harmonic Numbers,"[112] while Chinese scholarship simply terms them Small Numbers (*xiao shu* 小數). Their concern is to differentiate them from the calculations that begin with assigning the number 177,147 to *Huangzhong*, which yield Numbers (*da shu* 大數), as can be seen in List 3. Early Chinese texts refer to Harmonic Numbers only as *shu* 數, "numbers" or "regularities,"[113] but do refer to *da shu* when they use large numbers divisible by three, which make it easier to calculate the twelve tuning standards with no remainder.

The number 81 was significant in early Chinese cosmological numerology, and was used in various areas of early China. It was, for example, the largest number in the multiplication tables of the *Zhoubi Suanjing*.[114] By the Han, one of the major changes under the astronomical reform of Emperor Wu 武 (r. 141-87 B.C.E.), was the semantic change of the denominator or divisor (*rifa* 日法) in measuring the mean synodic month from 940 days to 81.[115] As Cullen explains the underlying reason behind this number: "This is the fourth

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[112] See, for example "nombre harmonique" for *shu* 數 Kalinowski, "Musique et Harmonie Calendaire."

[113] For more on *shu* see p. 85.

[114] Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing*, Needham Research Institute Studies 1 (Cambridge ; New York: Cambridge University Press, 1996), 29 (n.29), 83. I do not provide a composition date for the *Zhoubi Suanjing* due to its composite nature, and the "extremely varied degrees of interrelation" between its sections. Cullen, 138.

[115] Cullen, *Astronomy and Mathematics in Ancient China*, 29, 66. Christopher Cullen, "Motivations for Scientific Change in Ancient China: Emperor Wu and the Grand Inception Astronomical Reforms of 104 b.C," *Journal for the History of Astronomy* 24, no. 3 (August 1993): 185–203. Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems*, Scientific Writings from the Ancient and Medieval World (New York: Routledge, 2016)., 24-5. For further information on the use of the number 81 in the Grand Inception System see Nathan Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records: Shou Shih Li Cong Kao*, Sources and Studies in the History of Mathematics and Physical Sciences (New York: Springer, 2009), 63. For further information on the Grand Inception, Triple Concordance, and Quarter

power of 3, the number representing the principle of *yang* 陽, which pertains to the heavens as opposed to earth, to male as opposed to female, and to light as opposed to darkness."[116] Changing the divisor to 81 in this new Grand Inception Calendar (*Tai Chu Li* 太初曆), he argues, was an extension of the numerological associations to sound:

> The motive was not empirical, but was rather the wish to introduce into the system the cosmologically significant number 81, which…was not only the fourth power of the quintessentially yang number 3, but also represented the volume of the pitch pipe sounding the fundamental note of the scale, 'Yellow Bell' *Huangzhong* 黃鐘. Since music was held to embody the same cosmic order as was exemplified in the heavens, it would have been highly satisfying to see the motions of sun and moon governed by this number.[117]

The problem that arises when generating tuning standards using the *sanfen sunyi* formula, beginning at 81, is that it only allows the first five generated tuning standards to be expressed in integers:

Note	C (do) *gong* 宮		G (sol) *zhi* 徵		D (re) *shang* 商		A (la) *yu* 羽		E (mi) *jue* 角		**B (si)**[118]
Harmonic Number	**81**	-1/3	**54**	+1/3	**72**	-1/3	**48**	+1/3	**64**	-1/3	**≈ 42.667**

Remainder astronomical systems in the Han see Nathan Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy," *T'oung Pao*, Second Series, 55, no. 1/3 (January 1, 1969): 1–73.

[116] Cullen 2016, 25

[117] Cullen, *The Foundations of Celestial Reckoning.*, 34.

[118] On a seven-note scale, the sixth is called Altered *gong* (*bian gong* 變宮), and the seventh, F# (fa sharp), is called Altered *zhi* (*bian zhi* 變徵).

The author of FMTB List 2. was aware of this problem, and because the list continues beyond the five-note scale to twelve tuning standards, he rounds the harmonic number for the sixth generated tuning standard, *Yingzhong* (42.667) upwards (to 43). The remaining pitches must also be rounded upwards or downwards, depending on the remainder.[119]

Until the excavation of FMTB, evidence of an awareness and treatment of fractional remains was not apparent until the *Huainanzi* (139 B.C.E.), in which the formulations are rounded downwards (in this case, to 42).[120] Lastly, when using integers, the numbers of the twelve tuning standards cycle back only approximately to the starting point, ending at 80 instead of 81. In early China, the construction of pitches was close enough to cycles, so that it fit quite well with other sequences of generation like the five phases or the epochs of the calendar.[121] The approximation of the cycles only would be treated as a problem in need of solving only during the Western Han, with Jing Fang's reducing of this gap, known as a comma.[122]

List 3 contains the names of the twelve tuning standards, their Large Numbers, and the pitches that each generates. This is the first evidence of the use of Large Numbers before the three Western Han books: "Treatise on Tuning and Mathematical Astronomy" (*Lülizhi*

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[119] See also Cheng Shaoxuan 程少軒, *"Fangmatan jian,"* 259. n.8

[120] Kalinowski 2011, 122–23. *"yingzhong zhi shu sishier* 應鍾之數四十二." See "Celestial Patterns" chapter in the *Huainanzi*.

[121] For the use of cycles in the calendric system see Sivin, "Cosmos and Computation." and Cullen, *The Foundations of Celestial Reckoning*.

[122] Cheng Shaoxuan rightfully points out that if the generation of tuning standards using harmonic numbers was to continue beyond a single octave, while rounding results upwards or downwards, by the generation of the second octave, the entire temperament would be extremely inaccurate. The use of large numbers is far more accurate (although when generating the 60 tuning standards, Jing Fang resorted to rounding numbers several times). Cheng Shaoxuan 程少軒, "Fangmatan Jian," 260–61.

律歷志) in the *Shiji* and *Han shu,* and the "Celestial Patterns"*(*"Tianwen" 天文) chapter in the *Huainanzi*. Unlike the Harmonic Numbers cycle, the Large Numbers do not end where they began, and the thirteenth generated pitch would not have even been close to *Huangzhong*'s number (177,147 for *Huangzhong* vs 174,762 for *Zhishi*, the 13th tuning standard, which closed the octave).

The number for *Huangzhong* was selected so that all twelve tuning standards could be attained without remainders. In this process of pitch generation, the largest number must be divisible by 3 twelve times to avoid fractional remainders. The number 177,147, the initial Large Number, is 3 to the eleventh power (3^{11}).

List 3.

> 黃[鐘]十七萬七千一百卅七, [下林鐘]; 大呂十六萬五千 八百八十八, 下夷則; 大簇十五萬七千四百六十四, 下南呂; 夾鐘十四萬七千四百五十六, 下毋射; 姑先十 三萬九千九百六十八, 下應鐘; 中呂十三萬一千七十 二, [???]; 蕤賓十二萬四千四百一十六, 上大呂; 林鐘十一萬八千九十八, 上大族; 夷則十一萬五百九十 二, 上夾鐘; 南呂十〈四〉萬四千九百七十六, 上姑洗; 毋射九萬八千三百四, 上中呂; 應鐘九萬三千三 百一十二, 上蕤賓.

	Tuning Standard	Large Number	Generates
1	*Huangzhong*	177,147	*Linzhong*
2	*Dalü*	165,888	*Yizi*
3	*Dacou*	157,464	*Nanlü*
4	*Jiazhong*	147,456	*Wuyi*
5	*Guxi*	139,968	*Yingzhong*
6	*Zhonglü*	131,072	*???*[123]

[123] This 13th tuning standard should be *Zhishi* 執始, but the first time we encounter this name in received sources is Jing Fang's calculations in the *HHS*, and these date to the first century B.C.E. Since we do not yet know the origins of the names for Jing Fang's additional 48 tuning standards, this illegible character is at least an indicator that the 13th tuning standard, which signals the (inaccurate) cycling back to the origin and shows that the Pythagorean Comma existed and was named in the late Warring States period. See Part Two of this dissertation for more information on the comma.

7	*Ruibin*	124,416	*Dalü*
8	*Linzhong*	118,098	*Dacou*
9	*Yize*	110,592	*Jiazhong*
10	*Nanlü*	104,976[124]	*Guxi*
11	*Wuyi*	98,304	*Zhonglü*
12	*Yingzhong*	93,312	*Ruibin*

Third, the divination system in FMTB includes techniques using the five notes, the twelve tuning standards, and other correlative elements such as the five phases and *yin-yang*, dates, times of day, and celestial positions. Utilizing sound as a form of divination is not uncommon in early Chinese histories. In the *Shiji* "Book on Tuning Standards" ("Lü shu" 律書), it is associated with military affairs. Advisors interpreted subtle vibrations which they detected using pitch pipes in the ground. When they produced a sound, it indicated the location of an enemy and helped them assess the outcome of an upcoming battle. This type of divination does not seem to require calculation, but rather uses the pitch pipes as a predicting tool.

According to the *HHS* "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志), Jing Fang discusses the relationship between tuning standards and hexagram interpretation, and we are also told that Jing Fang changed his last name using a divination technique that employed the pitch pipes (see p. 64). There are those who argue that Jing Fang used hexagram interpretation only as an analogy to the generation of pitches, since they both show the undulation of *yin* and *yang* in the cosmos, but FMTB provides the first example of the use of tuning standards in hexagram divination. Following Cheng Shaoxuan's division of the FMTB, the section following the "Cosmic Board

[124] I'm uncertain why the transcription has the number 4 (四) in brackets here. The correct number doesn't require it.

Diagrams" is the "Divination Interpretations," which is subdivided into four sections, which he terms: "The Thirty-Six Animals Divination," "The Twelve Tuning Standards Divination," "Good and Bad Fate of the Twelve Tuning Standards," and "Hexagram Divination Unifying the Three Methods."[125]

For example, in the "Twelve Tuning Standards Divination," each bamboo strip is subdivided into four subsections: 1) a tuning standard and its manifestation as an object or notion followed by a *yi* 殹, which seems to have a grammatical function similar to a separator or exclamation mark.[126] This is followed by an interpretation similar in the fashion of the *xiang* (*xiang* 挂象) is similar to the sentence structure in Changes divination.[127] 2) a hexagram statement (*gua ci* 卦辭), composed of a series of (mostly) rhyming four characters. 3) ghosts and spirits linked with the tuning standard. 4) explanation of auspicious and inauspicious results for the situation at hand in relation to this tuning standard.

For example:[128]

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125 三十六禽占，十二律占，十二律吉凶，三合卦占. Cheng Shaoxuan 程少軒, "Fangmatan Jian," 282.

126 In the first divination example *Huangzhong* is correlated with a tone 音, but in the following divination defines the tuning standard *Dalü* 大呂, which is correlated with words (*yan* 言) in the opening "大呂；言殹." Each of the following tuning standards has different correlations, usually more than one. *Dacou* 大湊 is correlated with grieving (*you* 憂) and so on. Only *Huangzhong* is related directly to sound.

127 Cheng Shaoxuan 程少軒, "Fangmatan Jian," 294.

128 Following Cheng Shaoxuan's transcription and emendations. These are columns 260-261. Cheng Shaoxuan 程少軒, 294. See also Yan Changgui's transcription for variants:Yan Changgui 晏昌貴, "Tianshui Fangmatan Qin Jian Yi Zhong Rishu."

黃鐘：音殿. 貞在黃鐘，天下清明，以視陰陽，帝（帝）乃言+作[129]（作）之，分其短長，比于宮聲，意味[130]音尚，就[131]乃處之，十月再周[132]，復其故所。其奈[133]（祟）上□[134]，先殤[135](?)。卜疾人，三禺（遇）黃鐘，死。卜事君，吉。

Huangzhong[136]: tone. divining on *Huangzhong*, all under heaven clear and bright, by observing *yin* and *yang*, Huang Di[137] then gives rise to it, dividing its short and long, compare with the position of the *gong* note, meaning the most important tone, return and locate it,[138] tenth month and then it revolves once more,[139] returning to its origin (?). Its ghost rises □[140], an ancestor who died before reaching adulthood (?), divining for an ill person, if you [cast and] strike *Huangzhong* three times[141], then death [will occur]. If you are divining about serving the ruler, it is auspicious.

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[129] Yan Changgui 晏昌貴 glosses this character as 誂, meaning hastily or suddenly. Yan Changgui 晏昌貴, "Tianshui Fangmatan Qin Jian Yi Zhong Rishu," 33.

[130] Yan Changgui : 以為. Ibid.

[131] Yan Changgui : 久. Ibid.

[132] Yan Changgui: 唐. Ibid.

[133] Should be 祟

[134] Yang Changgui : 商. Ibid.

[135] Yan Changgui: 陽. Ibid. Chen Shouxian argues that the radical on the left is 歹, thus the character should be 殤 and not 陽. *Shang* 殤 could also refer to the name of the ghost. Cheng Shaoxuan 程少軒, "Fangmatan Jian," 296.

[136] Referring to some kind of hexagram depicting *Huangzhong*.

[137] Cheng argues that this Di refers to Huang Di, based on later mythologies that relate Huangdi with the creation of the tuning standards. Cheng Shaoxuan 程少軒, "Fangmatan Jian," 296.

[138] This could be discussing an entire cycle from one *Huangzhong* to the next, and relating it to the calendar in the next quadruplet.

[139] Establishing the Branches (?) (*Jian hai* 建亥) also known as Establishing the Dipper (*Dou jian* 斗建) calendric system is used in other sections of FMTB. According to this system, the year began on the tenth month. Cheng Shaoxuan 程少軒, "Fangmatan Jian," 296.

[140] Indecipherable character.

[141] Referring to cleromantic divination techniques, in which an object is cast (*tou* 投) and the results are interpreted, much like the use of yarrow sticks in Changes interpretations. Cheng Shaoxuan 程少軒, "Fangmatan Jian," 297. For an elaboration on the system and additional examples of divination using tuning standards in FMTB see Kalinowski, "Musique et Harmonie Calendaire," 133–34.

The information that FMTB provides regarding the numerological use of tuning standard ratios in divination is invaluable. The text does not discuss the use of instruments or the need for players, nor does it refer to musical compositions. It discusses only tuning standards and the music system that produces them. They are quantitative, reflecting cosmic processes, and therefore lend themselves well to predictions.

Another divination technique used quantitative tuning standards. According to this method, if one casts for a tuning standard, but the result is not accurate, the number is rounded to the nearest tuning standard, according to the time of day:

> 凡投黃鐘不合音數者是謂天絕紀殴數有六十六旦從六十八夕從六十四數七十五占七十六數有卅四占卅二
>
> Whenever casting for the pitch standard does not match the harmonic number, this is called "heaven severs its support cords." If the number is 66, in the daytime use 68 and at nighttime use 64. If the number is 75, make the divination for 76. If the number is 44, make the divination for 42.[142]

Kalinowski adds the following: "It should be noted that the largest number obtained by adding the number of the day-stems, the day-branches, and the time periods of the day is 27 (9+9+9), which multiplied by 3, equals the harmonic number of the lowest pitch standard (Yellow Bell, F, 81). Likewise, the smallest number thus obtained is 14 (5+4+5), which, multiplied by 3 equals the harmonic number of the highest pitch standard (Yingzhong, E, 42)."

The techniques shown above, which relate tuning standards to hexagram interpretation, are a precursor of the type of divination likely conducted by Jing Fang, which

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[142] Harper and Kalinowski, *Books of Fate and Popular Culture in Early China*, 474.

I briefly discuss in Part Two. Jing Fang is presented as an expert in the *Changes*, weather divination, and musical temperament. The connections among all three are evident in FMTB.

Comparing the Zeng bells, FMTB, and the "Da Siyue" section of the *Rites of Zhou* shows that tuning, in which I include the selection of specific pitches and scales, was an important aspect of the theory of sound in early China, both inside and outside the musical realm. The Zeng inscriptions present a musical system that includes various elements not mentioned or used in later textual accounts, deliberately constructed to include various options for generating a scale on each of the twelve tuning standards, to be performed in ritual music. The "Da Siyue" section describes how those who oversaw the performance of ritual music chose scales that would portray the cosmological scheme of rising *yang* and falling *yin*. However, they do not give any special significance to the numerical expression of tuning standards. FMTB shows yet again that quantitative tuning standards had great significance for divination practices, because sound represented cosmic harmony and the ability to quantify sound implied control over the outcome of actions. In addition, the cosmological significance of tuning is apparent through mythologies that explain the creation of tuning standards, independent from tales about the creation of music.

In conclusion, we can say that in early China, tuning was important both inside and outside of the musical realm, and from around the fourth century B.C.E. onwards, it indicated the state of the cosmos. Ritual music was one translation of this cosmic situation, divination was another, astronomy was yet another, and weights and measures, were still another. It is the combination and amalgamation of all these in the Han state that calls attention to the cosmic notion of sound measurement and to its dominance in early China. The main techniques and ideas were already in practice by the mid-third century B.C.E.

Part Two

MIND THE GAP: JING FANG AND COSMOLOGICAL TUNING IN THE WESTERN HAN

This chapter bridges the gap between the technical and the cosmological aspects of sound in the Western Han, by examining the case of Jing Fang 京房 (78-37 B.C.E.) and his musical theory of sixty tuning standards (*liushi lü* 六十律), presented in the "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of the Later Han*.[143] I argue that concepts of *yin-yang*, and *qi*, as well as the ideal of the supremacy of numbers, caused literati like Jing Fang to conceptualize sound as a tangible, useful component, a vibrating *qi* attuned to the natural order of the cosmos. I also argue that Jing Fang's theory of the sixty tuning standards is an exemplar of how tuning and sound were laden with cosmological meaning. Accordingly, these small components (pitches, scales, measures) were what gave music its meaning.

In this way, we can begin to appreciate some of the technical building blocks behind the concept of "correcting music" (*zheng yue* 正樂) in this period. In a sense, music in the Han should be considered a product of tuning. Sound and tuning were tools which legitimized the ruling dynasty, according to the accepted standard. In order to use it properly in music and in ritual, one had to be certain that one's chosen pitch measures for each constructed scale were accurate and compatible with the current calendar and the seasons.

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[143] Lüli zhi 律歷志, hereafter: LLZ . History of the Later Han, hereafter: *HHS*.

This bolsters the larger argument of this dissertation: the goal of producing accurate pitches and calculations did not necessarily coincide with that of making music more beautiful to the listener. Its goal was to obtain the keys to the cosmos—musically and cosmologically.

This chapter is divided into three sub-sections. Section 1 introduces Jing Fang, his biography and accomplishments. Section 2 is an annotated translation of the first part of the *HHS* LLZ. Finally, section 3 is an analysis of the acoustical and cosmological aspects of the treatise, including Jing Fang's calculations of the sixty tuning standards, his motivation for reducing the Pythagorean comma, and the underlying cosmological concepts that drove his innovation.

Section One: Jing Fang's Biography

Jing Fang 京房 (78-37 B.C.E.) was a specialist in the art of interpreting the *Changes* (*Yi* 易) who served the Western Han court during the reigns of emperors Yuan 元 (r.48-33 B.C.E.) and Cheng 成 (r.33-7 B.C.E.). Because he is one of the few dramatis personae in the Han histories who are characterized with a detailed, strong association between cosmology and sound, he is especially significant for the study of early Chinese musical thought. His calculations and explanations are musicologically important and enrich the musical world of Han textual sources. I will address these in depth in the analysis section of this chapter.

With regard to tuning, Jing Fang is known for four accomplishments: 1) Expanding tuning computations from twelve to sixty pitches within a single octave. The calculations also attest to his awareness of a gap between the first and thirteenth generated pitch, and his intention to reduce it significantly. 2) The application of the pitches and their derived musical modes to the days of the luni-solar calendar. 3) the construction of a tuning instrument called the *zhun* 準, or Equalizer, which greatly facilitated practical use of his calculations, but was lost soon after his time. 4) The application of this knowledge to a ritual called Observing *Qi* (*hou qi* 候氣), which served to ascertain a correct correlation between cosmic *qi* and the musical modes each month of the year through the vibration of pitch pipes planted in the earth. These advancements in tuning and cosmology join his achievements as a diviner and advisor to the Han court.

Biographical details on Jing Fang are sparse. Most of the available information appears in the "Biographies of Sui, the two Xia Marquessses, Jing, Yi, and Li" (*Sui liang Xia Hou Jing Li Zhuan* 眭兩夏侯京翼李傳) in the *Han shu* and the *HHS* LLZ. The

bibliographical treatise (*Yiwen zhi* 藝文志) in the *Han shu* mentions two titles bearing Jing Fang's name; both are commentaries on the teachings of *Changes* expert Meng Xi 孟喜 (n.d.). Their titles are *Jing Fang on Master Meng* (*Meng Shi Jing Fang* 孟氏京房)[144] in eleven chapters, and Jing Fang on *Master Meng's "Disasters and Abnormalities"* (*Zaiyi Meng Shi Jing Fang* 災異孟氏京房) in sixty-six chapters. Also, fragmented quotations of a work titled a *Commentary on the Book of Changes by Jing Fang* (*Jing Fang Yi Zhuan* 京房易傳) are scattered throughout the *History of the Former Han* and the *History of the Later Han*, mainly in treatises on the five phases (*wu xing* 五行).[145]

The *Han shu* mentions two men named Jing Fang (京房). The direct information on the elder Jing Fang, hereafter Jing Fang (I), boils down to his political office and his expertise. Jing Fang (I) served as Superior Grand Master of the Palace (*Taizhong Dafu* 太中大夫), a distinguished position as counselor to the emperor.[146] He was also appointed governor of Qi Commandery (齊郡). His expertise, like that of the younger Jing Fang, hereafter Jing Fang (II) was in the interpretation of the *Changes*. He was a teacher of Liangqiu He 梁丘賀, who later established one of the four traditions of interpretation of the *Changes*. Indirect information suggests that he died sometime before the reign of Emperor Xuan (r. 74-48 B.C.E.). Hulsewé concluded from the sparse evidence that Jing Fang (I) lived sometime

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[144] Despite the grammar, which requires reading this as "Master Meng's [commentary on] Jing Fang," Meng Xi lived prior to Jing Fang, and so the title must refer to the latter's interpretation of the former.

[145] Hulsewé suggested the *Jing Fang Yi Zhuan* was the work of another expert on the *Changes* by the same name, who lived around the years 140 to 80 B.C.E. Loewe argues against this. Both positions are presented below.

[146] Charles O Hucker, *A Dictionary of Official Titles in Imperial China* (Taipei: Southern Materials Center, 1985), 477.

between 140-80 B.C.E.,[147] and attributed the *Jing Fang Yi Zhuan* (京房易傳), to him. Loewe, on the other hand, attributed it to Jing Fang (II). Hereafter, I discuss only the work attributed to Jing Fang (II).

Jing Fang's style name (字) was Junming (君明). His place of origin was Dunqiu 頓丘 county, then located on the western border of Dong Commandery 東郡 in present-day Henan. He functioned in several official positions, and in 45 B.C.E. was appointed Gentleman of the Interior (*Langzhong* 郎中), an honorary position in the imperial court.[148] Along with his expertise in interpreting the *Changes*, he was a diviner with a good record of realized prognostications, as well as a political figure with an agenda, known for strongly advocating meritocratic promotions. His political activities, which directly targeted powerful officials such as the eunuch Shi Xian 石顯 and Wu Lü 五鹿, destabilized his position in court, leading eventually to his execution at the age of forty-one.

Jing Fang's *Han shu* biography details several occasions in which he counseled Emperor Yuan on identifying disloyal officials. Because of his excellent record in prediction, the emperor first consulted Jing regarding an uprising of the Western Qiang 西羌 people, which was followed by a solar eclipse. Later, Jing Fang approached the emperor, warning him once more against the quality of his officials. Both times Jing argued that the calamities and strange phenomena afflicting the realm were a result of the current political system, which took flatterers for men of talent. He advocated meritocratic selection and was

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147 A. F. P. Hulsewé, "The Two Early Han 'I Ching' Specialists Called Ching Fang 京房," *T'oung Pao* 72 (1986): 161–62.

148 This position indicates that one is awaiting an official appointment. Hucker, *A Dictionary of Official Titles in Imperial China*, 301.

permitted to create a method and examine the achievements of government officials, and choose disciples who would carry it out after him. In his warnings, he relied on the omens detailed in the *Springs and Autumns of Master Lü* (*Lüshi Chunqiu* 呂氏春秋) and urged the emperor to heed the strange natural phenomena and the state of society as signs that Shi Xian and his like were leading the dynasty to its demise. An abridged version of this account also appears in the chapter "Advice and Admonitions" (*Gui Zhen* 規箴) in the fifth-century compilation *A New Account of the Tales of the World* (*Shi Shuo Xin Yu* 世說新語).

The *Han shu* paints a picture in which his conflict with Shi Xian and Wu Lü cost Jing his life. His third warning, in the form of several sealed memorials to the emperor, was sent once he had been removed from the capital and appointed Governor of Wei commandery (*Weidu Taishou* 魏郡太守). His fear for his life and admonitions against the deception and excess power of the high officials naturally grew much stronger. To these he added detailed predictions of severe weather and cosmological portents, which could be lifted only with his return to court.

The last section of the *Han shu* biography seems to recount a different perspective of Jing Fang's story, focusing on his relationship with Zhang Bo 張博[149] and their attempt to implement Jing's method of examining the achievements of officials within the court. The important addition is Shi Xian's reaction to their plans: afraid for his position, he waited for Jing Fang's removal from the capital before charging him and Zhang Bo with colluding and

[149] The biography explains that Zhang Bo was related to Emperor Yuan′s brother, Liu Qin, ling of Huai Yang (淮陽憲王).

slandering the government and emperor. Following this accusation, Jing Fang and Zhang Bo were both executed.[150]

Jing Fang's original sir name was Li (李). He changed it to Jing (京) after using a technique called "Inferring from the Pitch pipes" (*tui lü* 推律).[151] Hulsewé suggested that apart from the possible cosmological benefits of this name change, Jing Fang perhaps "wanted to share the fame" of the other well-known interpreter of the *Changes* who bore the same name (i.e Jing Fang (I)).[152] While this is a plausible argument, taking someone else's name could have proven dangerous, and the agenda of the *HHS* LLZ seems to suggest a cosmo-musical reason behind the process.

This name change is intriguing because it points to a direct link between divination and sound. Unfortunately, his biography does not provide details on the technique or the ideas behind it. There is also the larger question of whether there was a regular divination practice involving pitch pipes, or whether this was a unique instance mentioned for its oddity.

The Qing commentator Shen Qinhan (沈欽韓 1775-1831) turned to two early accounts and an additional account from the southern Song dynasty (宋 1127-1279), which comment on the creation of clan-names. The first early source is part of the chapter "Family and Personal Names" (*Xingming* 姓名)[153] in the Eastern Han compilation *Discussions in the*

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[150] Ban Gu 班固, ed., *Han shu* 漢書, vol. 4 (Beijing: Zhonghua shuju 中華書局, 1964), 160–67.

[151] Ban Gu 班固, 4:3167.

[152] A. F. P. Hulsewé, "The Two Early Han 'I Ching' Specialists Called Ching Fang 京房," 162.

[153] Following Tjan Tjoe Som in Tjoe Som. Tjan, *Po Hu T'ong : The Comprehensive Discussions in the White Tiger Hall*, vol. 2 (Leiden: Brill, 1949), 24.

White Tiger Hall (*Bai Hu Tong* 白虎通),[154] and the second is Wei Zhao's (韋昭 204–273 C.E.) third century commentary to the "Discourses of Zhou" (*Zhou yu* 周語) chapter in *Discourses of the States* (*Guoyu* 國語). Both sources were written after Jing Fang's time. The "Discourses of Zhou" chapter has been dated to the fifth century B.C.E.,[155] but the main text provides no information on musical use of instruments in clan-name choices. Wei Zhao comments:

> 司商掌賜族受姓之官. 商,金聲, 清. 謂人姓, 吹律合定其姓名也.[156]
>
> The Zhang[157] Director office oversees giving clan names, and handing family-names. The *shang* [note] is metal, its sounds are clear. In order to tell people's clan-names, they blew air into pitch pipes to unify and fix their family and personal names.

The *Discussions in the White Tiger Hall* can be dated at least to the first century, if not later.[158] However, two ideas in the "Family and Personal Names" chapter may relate to the method whereby Jing Fang changed his surname. First is the argument that *qi* plays a crucial

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[154] Hereafter *Discussions*

[155] Loewe, *Early Chinese Texts*, 264.

[156] Xu Yuangao 徐元誥, *Guoyu Jijie*, 24.

[157] Possibly refers to the second note in the Five-Note scale (*shang* 商), but the *Guoyu* Commentary states *shang* 商 should be read as *Zhang* 章, and the title refers to the Music Director (*ranze Si Yue zhe, wei zhi Si Zhang ... Yuangao an: "chui lü dingxing, nai Siyue zhi shi."* 然則司樂者，謂之司章……元誥按：『吹律定姓，乃司樂之事』. Xu Yuangao 徐元誥, 24.

[158] *Han Shu Bu Zhu* 漢書補註, vol. 2 (Beijing: *Shumu wenxian* 書目文獻, 1995), 1374.The association to the *Discussions* is logical, since the *HHS* discusses several events leading to a compilation of a similarly titled text (*Bai Hu Tong De Lun* 白虎通德論) in the first century, and attributes its editing to Ban Gu 班固 (32-92 C.E.). Since Jing Fang′s name change anecdote came from Ban Gu′s other work, the *Han shu*, the two would seem to be ideal complimentary sources. But the authenticity of the received version of the *Discussions of the White Tiger Hall* is uncertain, and we are arguing its reflection of knowledge and attitudes of the Han court in the first century must be selective. Loewe discusses the inconsistencies in title, number of chapters, and content at great length in Loewe, *Early Chinese Texts*, 347–56. Also see Tjan, *Po Hu T'ong : The Comprehensive Discussions in the White Tiger Hall*, 2:166–78.

role in determining one's clan-name. When a family-name 姓 is given, it carries a heavenly *qi*.[159] The second is that this *qi* is divided through the five notes to somehow determine one's last name. Below I follow Tjan's translation:

> 姓所以有百者何？以為古者聖人吹律定姓，以紀其族。
>
> Why are there one hundred clan[160]-names? Anciently the Sages by blowing the musical pitch-pipes fixed the clan names, and thereby registered the [different kinds of] kindred.
>
> 人含五常而生，聲有五音，宮、商、角、徵、羽，轉而相雜，五五二十五，轉生四時，故百而異也。氣殊音悉備[161]，故殊百也。
>
> Man is born with the Five Constant [Virtues] in him. There are five principal tones: kung, shang, chüe, chih, and yu, which, combining together five by five, make twenty-five [tones], and further give birth to the four seasons. With the [four] different climates and the [twenty-five] various tones the completion is obtained. Therefore, the clans are diversified into one hundred.[162]

The last source presented by Shen Qinhan is a Song-dynasty compilation titled *Gujin Hebi Shili Beiyao* 古今合璧事類備要. Although it gives us the most specific information regarding names and tones, it was written over a millennium after the Western Han, and provides nothing directly pertinent to Jing Fang, but could indicate later use of this custom. None of these sources provides information on changing names, but only on giving names. Moreover, Jing Fang's method for changing his name included "inferring from the pitch

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[159] "姓生也，人所稟天氣所以生者也" Tjan translates: *"Hsing* 'clan-name' means *sheng* 'to live'. Man receives life by the grace of Heaven's fluid [*qi*]." Tjan, *Po Hu T'ong : The Comprehensive Discussions in the White Tiger Hall*, 2:579.

[160] Tjan translates *xing* 姓 as "clan-names," while I refer to it as "family-names," and *zu* 族 as "clan-names."

[161] This part is particularly difficult, and Tjan's translation seems to bypass it.

[162] Tjan, *Po Hu T'ong : The Comprehensive Discussions in the White Tiger Hall*, 2:579.

pipes" (*tui lü* 推律), and not "blowing into the pitch pipes" (*chui lü* 吹律). Still, the sources are valuable for the ideas at play. There seems to have been a mythology around the creation and dissemination of family names, which incorporated the use of the Five-notesand the Pitch pipes as determiners, by their function as conductors of heavenly *qi*.

This mythology fits quite well with Jing Fang's name change, and perception of sound and the cosmos as it appears in the more detailed *HHS* account. The method may or may not have been an active form of divination in the Western Han, but one can see why it is associated with Jing Fang. Sound was his territory of expertise, and this was a striking example of it.

Jing Fang's background in tuning computations is unclear, but seems to relate more strongly to cosmology than to musical performance. He is described tersely only as "fond of tuning computations (鍾律 bell standards), and proficient in tones and notes,"[163] or having knowledge of the "Tones of the five notes, and the numerical relations of the six modes."[164] He is not presented as a music master. There is no mention of the content of musical performance, and there are no references to any instrument he played (the instrument he built was for tuning purposes). The available information suggests his expertise lay in accurate and detailed computation of tuning standards, modes, their division according to the calendar, and using these in prognostications.[165]

The *History of the Jin Dynasty* (*Jin Shu* 晉書) tells us that the Eastern Han scholar Cai Yong (132-192 C.E.) pursued Jing Fang's calculations of the sixty tuning standards, which

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[163] Ban Gu 班固, *Han shu* 漢書, 4:3160.

[164] Two sets of six-*lü* each, divided into six *yang* and six *yin* tuning standards and the musical modes constructed using each of these base tones. These are discussed in more detailed on p. 87.

[165] What we would today frame as part of the larger field of acoustics.

had been lost by his time. Cai Yong retrieved the text, but was unable to apply it. It also tells us that contemporary acousticians (*yinjia* 音家) used this text, but the sixty tuning standards were not applied to musical performance (*Liushi lü zhe wu shi yu yue* 六十律者無施於樂).[166]

In the *HHS* LLZ, translated below, Sima Biao 司馬彪 (ca. 240- ca. 306 C.E.)[167] argues for Jing Fang's significance by virtue of his detailed work on tuning standards (*lü* 律). Liu Xin 劉歆 (46 B.C.E. – 23 C.E.) was the first to compile a full treatise on tuning standards and mathematical astronomy ("Lüli zhi" 律歷志), currently transmitted as a chapter of the *Han shu*,[168] but it did not contain as much detailed information as Jing Fang's treatise. Moreover, Jing Fang's inclusion in the *HHS* chapter, though he was a Western Han persona, is attributed to the practical uses of his work by astrological and astronomical imperial offices. Following this justification, the author of the treatise laments the loss of this knowledge in the Eastern Han and provides two specific cases nearly two hundred years apart, in 84 C.E. and in 177 C.E., when officials wished to advance their tuning methods, but had to cease their efforts due to lack of information.[169]

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[166] Fang Xuanling 房玄齡, *Jin Shu* 晉書, vol. 1 (Beijing: Zhonghua Shuju 中 華書局, 1974), 483.

[167] The Jin dynasty historian and expelled royal member, Sima Biao, is the compiler of the "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *Continued Book of Han* (*Xu Han shu* 續漢書). This treaty was then used in the *HHS*. However, Sima Biao did not write the treatise on the pitch pipes, but used former version written and edited by Cai Yong 蔡邕 and again by Liu Hong 劉洪 (fl.180), both Eastern Han figures. We can attribute the justification of Jing Fang's inclusion in the treatise to any of the three. See B. J. Mansvelt Beck, *The Treatises of Later Han: Their Author, Sources, Contents, and Place in Chinese Historiography*, Sinica Leidensia, vol. 21 (Leiden ; New York: E.J. Brill, 1990), 61.

[168] The *Shiji* also includes a chapter on tuning, however, its authenticity, and that of the treatise on musical performance (*Yue Shu* 樂書) has been questioned. See Mansvelt Beck, 56. and Martin Kern, "A Note on the Authenticity and Ideology of Shih-Chi 24, 'The Book on Music,'" *Journal of the American Oriental Society* 119, no. 4 (October 1, 1999): 673–77.

[169] See the annotated translation of these accounts in the second section of this chapter.

This in no way disqualifies the relation between cosmological tuning and ritual musical performances. We have several indications that correct tuning was related to music performances throughout the Western and Eastern Han,[170] and even Jing's indirect connection to music is evident when he is questioned about his tuning technique at the Music Bureau (*yuefu* 樂府).[171] The treatise also mentions that in 74 C.E. Nanxuan was tested on the method of the *zhun* in order to fix and harmonize musical instruments (*zhutiao yueqi* 主調樂器).[172] However, since Nanxuan failed and did not win a post, it is possible that, despite a great will to tune instruments according to cosmological computations, this simply did not happen, at least in the Eastern Han.

The third of Jing Fang's musical accomplishments is the initiation of the practice called Observing *Qi*. This practice took musical intervals and converted them into time intervals throughout the solar year.[173] Although it seemed to produce pleasing results in his case, few after him managed to recreate it successfully.[174] Nevertheless, the practice continued and was accepted well into the Ming dynasty. Regardless of the credibility of this type of ritual in modern times, to Jing Fang it was a practical implementation of his work, which tested the veracity of his cosmological theory.

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[170] These are discussed in chapter 3.

[171] See section 2 for translation.

[172] I translate *tiao* 調 in its early meaning "to harmonize." I have not seen examples of it being used as "to tune" prior to the Six Dynasties.

[173] Not unlike the translation of the movement of cosmic bodies into time periods via water clocks (space was measured through time).

[174] Yilong Huang and Chih-ch'eng Chang, "The Evolution and Decline of the Ancient Chinese Practice of Watching for the Ethers," *Chinese Science* 13 (1996): 83.

Section Two: Annotated Translation of the First Section of the "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of The Later Han*

The treatise above is divided into six main sections:[175]

a) The supremacy of numbers since mythological times and their practical uses.

b) A short review of tuning and astronomy from the Qin to the Western Han.

c) Jing Fang's theory of cosmological tuning, his crafting of a tuner to facilitate measuring pitches, followed by a justification of Jing Fang's inclusion in this chapter.

d) Technical explanations, calculations, and the need for Jing Fang's tuner.

e) On the subtlety of tones: two examples from the Eastern Han.

f) The ritual of Observing *Qi*: theory and instructions.

[175] The reader can click each title to skip to the desired section.

律準　候氣 Tempering the *Zhun* and Observing *Qi*

古之人論數也，曰「物生而後有象，象而後有滋，滋而後有數」。

What the ancients said about numbers was: "phenomena are born, and then have counterparts.[176] Once they have counterparts, they sprout. Once they sprout, they have number."[177]

然則天地初形，人物既著，則筭數之事生矣。記稱大橈作甲子，　隸首作數。

That being so, when Heaven and earth had first taken shape, and men and things were already manifest, the calculation of numbers began. The records state that Da Rao[178] created the ten heavenly stems and twelve earthly branches, and Li Shou[179] created numbers.

二者既立，以比日表，以管萬事。夫一、十、百、千、萬，　所同用也；律、度、量、衡、歷，其別用也。

When both (the sexagenary cycle and numbers) were established, they were applied to the gnomon and the management of the myriad affairs. One, ten, hundred, thousand, ten

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[176] Schafer's translates *xiang* 象 as counterpart, especially fitting in this case despite his reliance on medieval, rather than early, texts, due to its connotation to a "universe conceived as a resonant system of celestial and terrestrial correspondences." *Pacing the Void: T'ang Approaches to the Stars* (Warren, Conn: Floating World Editions, 2005), 5. He defined celestial *xiang* as kind of doppelgangers to earthly events (55-56). He also accepts "Image," or "simulacra" as possible translations to *xiang*, but in my view "counterpart" emphasizes a relationship of extension, rather than reflection between celestial and earthly phenomena. Such is the case with sound and tuning.

[177] This sentence also appears in the *Zuozhuan* 左傳 in a discussion between Lord Hui and Han Jian. In that context, Han Jian about divination. For the English translation see Stephen W Durrant, Wai-yee Li, and David Schaberg, *Zuo Tradition = Zuozhuan: Commentary on the "Spring and Autumn Annals,"* 2016, 327.

[178] Da Rao is the name of a mythological character said to be a minister of the Yellow Emperor. He is said to have created the sixty-year cycle.

[179] The mythological scribe (*shiguan* 史官) for the Yellow Emperor, Li Shou is said to have created the counting of numbers. In the *HYDCD*, he is portrayed as a character named Li Shou, although this may simply mean "Head of the Clerks."

thousand are used in the same way. Musical modes, degrees, weights and measures, and the calendar are among their other uses.

故體有長短，檢以度；物有多少，受以量；量有輕重，平以權衡；聲有清濁，協以律呂；三光運行，紀以歷數：然後幽隱之情，精微之變，可得而綜也。

Thus, bodies are longer or shorter, measured in degrees. Objects can be many or few, encompassed by measures. Weight can be light or heavy; it is leveled by the weight and beam of the steelyard. Pitch can be clear (high) or turbid (low)[180]; it is tuned[181] by the tuning standards. The motion of the sun, moon, and planets is recorded using calendric constants. After doing this, one can sum up the hidden realities and the subtle transformations.

漢興，北平侯張蒼首治律歷。孝武正樂，置協律之官。至元始中，博徵通知鍾律者[182]，考其意義，羲和劉歆典領條奏，前史班固取以為志。

At the start of the Han dynasty, the Marquess of Beiping, Zhang Cang,[183] first administered tuning standards and astronomy.[184] Emperor Wu rectified musical performance, and set up

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[180] I have reservations about the translation of 清濁, and until a better definition is available, I prefer to leave it as "clear" and "turbid" in translation of early texts. More on this in chapter 1.

[181] The word *xie* 協, which I translate here as *to tune*, in fact means *to harmonize*. Since there are a few words that are translated as "harmonize," I chose to treat its practical aspect – the tuning of the pitch pipes and the standardization of weights and measures accordingly – to avoid misinterpretation. The determination of the correct pitch for the Yellow Bell pitch pipe is the most poorly understood activity in this process of tuning/harmonizing.

[182] *Zhonglü* 鍾律 is translated as tuning, temperament. Originally referred to the twelve *lü* in a bell set and later acquired the general meaning of temperament.

[183] Zhang Cang 張蒼(appointed Marquess of Beiping 北平 in B.C.E. 201) was prominent in the establishment of the Western Han calendar and the dynasty's Zhuanxu 顓頊 astronomical system of standard weight and measures (also used in the Qin). Zhang held the office of Imperial Chancellor (*chengxiang* 丞相) for fourteen years, and wrote additional works on *Yin Yang* and the calendar. For Zhang Cang's biography See Michael *Loewe, A Biographical Dictionary of the Qin, Former Han and Xin Periods 221 B*C - AD 24 (Leiden; Boston; Köln: Brill, 2000), 675–676.

[184] Following Sivin (Sivin, *Granting the Seasons*, 229.)

an office for Harmonizing the tuning standards.[185] Between 1 and 5 C.E., [Emperor Ping] broadly recruited experts on (music) temperament to examine the meaning and significance of tuning the musical modes. The Astrologer[186] Liu Xin was in charge [of the research] and made it into a memorial. Ban Gu used this memorial as a chapter in the *History of the Former Han.*[187]

而元帝時，郎中京房[房字君明] 知五聲之音，六律之數。上使太子太傅韋玄成、宇少翁，諫議大夫章，雜試問房於樂府。房對：「受學故小黃令焦延壽。六十律相生之法：

In the reign of Emperor Yuan, the Gentleman of the Interior[188] Jing Fang, styled Junming, knew the tones of the Five-notesand the numerical relations of the six modes. The emperor ordered the Grand Mentor of the Heir Apparent, Wei Xuancheng, styled Shaoweng, and the Grand Master of Remonstrance Zhang to test Fang with miscellaneous questions in the

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185 This seems to be directly related to the regulations of weights and measures. Li Yan Nian 李延年 was, perhaps, the person in charge of establishing the length of the Yellow Bell pitch pipe under Emperor Wu 武帝 (r.141–87 B.C.E.), according to which all other measurements were determined.

186 Liu Xin (46 B.C.E.-23 C.E.) was not presented with the title *xihe* 羲和 until 5 C.E. under emperor Ping 平帝（r. 1-5 C.E.). Hucker marks this title with a question mark, translating it as Astrologer (?) or "on the staff of the Grand Astrologer (t'ai shih ling) (?)"

The son of Liu Xiang 劉向, Liu Xin, was responsible for some of the most important records of the Han Dynasty. As for reforms in the pitch pipes and calendar, Loewe notes: "Commissioned to set out the prescriptions for musical instruments and the pitch pipes, Liu Xin produced a lengthy and detailed document, which concerned cosmology, astronomy, music, numerical calculation, and the system of weights and measures. Above all he set forth the principles of the San Tong Li 三統曆 for regulating the calendar. Much of this essay, which bore on a consideration of China's pre-imperial history, was included in the *Han shu*, as were his retrospective interpretations on some fifty strange occurrences of nature that were reported for the Spring and Autumn period. His interpretations were markedly different from those offered by Dong Zhongshu 董仲舒 and Liu Xiang, which were based respectively on the Gongyang and Guliang traditions." For Liu Xin's biography see *Loewe, A Biographical Dictionary of the Qin, Former Han and Xin Periods 221 BC - AD 24*, 383–386.

187 This refers to the "Treatise on Mathematical Harmonics and Astronomy" (律曆志) in the *History of the Former Han* (漢書)

188 Hucker p. 301, entry 3565.

Music Bureau. Fang replied: "I was taught by the former Palace Attendant[189] Jiao Yanshou. The method of mutual production of the sixty pitches is as follows:

以上生下，皆三生二，以下生上，皆三生四，陽下生陰，陰上生陽，終於中呂，而十二律畢矣。中呂上生執始，執始下生去滅，上下相生，終於南事，六十律畢矣。

[When] up produces down, then in all cases threes produce twos. [When] down produces up, then in all cases threes produce fours. When the *yang* goes downward it produces *yin*, and when *yin* goes upward it produces *yang*. When the cycle ends at the *Zhonglü* pitch, the twelve pitches are thus complete! When *Zhonglü* goes upwards it produces [the tone] *Zhishi*. When *Zhishi* goes downwards it produces [the tone] *Qumie*. Upwards and downwards they produce each other. Culminating at *Nanshi*. The sixty pitches are thus complete!

夫十二律之變至於六十，猶八卦之變至於六十四也。宓羲作易，紀陽氣之初，以為律法。

This transformation of the twelve pitches reaching sixty is like the transformation of the eight trigrams' reaching sixty-four. Fu Xi wrote the *Book of Changes* and recorded the beginnings of the *yang qi*. From this came the method of the modes.[190]

建日冬至之聲，以黃鍾為宮，太蔟為商，姑洗為角，林鍾為徵，南呂為羽，應鍾為變宮，蕤賓為變徵。此聲氣之元，五音之正也。

He established as the sound corresponding to the winter solstice the Yellow Bell [pitch] as the *gong* note, [then] Grand Manifold as *shang*, Maiden Cleanliness as *jue*, Forest Bell as *zheng*, Southern Pipe as *yu*, Responsive Concentration as the transformed *gong*, and Luxuriant Guest

[189] The title *xiao huang men* 小黃門 usually referred to high-ranking Eunuchs.

[190] In other words, the author is saying, the order of the hexagrams beginning with pure *yang* was Fuxi's conscious precedent for the 12 + 60 pitches.

as the transformed *zheng*. This is the beginning of the correspondences of sounds and *qi*, and is the correct ordering of the five tones.

故各(終)[統]一日。其餘以次運行，當日者各自為宮，而商徵以類從焉。禮運篇[191]曰『五聲、六律、十二管還相為宮』，此之謂也。

Each [of the seven sounds] corresponds to a single day. The remainder [of sounds] go through the cycle in sequence. The ones that correspond to [a given] day are spontaneously [in the] *gong* [position], and *shang* and *zhi* then take over, according to their categories.[192] When in the "Circulation of Rites"[193] chapter it is said: "The five sounds, six pitches, and twelve pipes circle back to *gong*," it refers to this.

以六十律分期之日，黃鍾自冬至始，及冬至而復，陰陽寒燠風雨之佔生焉。於以檢攝群音，考其高下，苟非草木之聲，則無不有所合。虞書曰『律和聲』，此之謂也。」

The sixty modes divide the days within a period. [The duration of] Yellow Bell is from one winter solstice until it repeats at the following solstice, giving rise to divination of the *yin* and the *yang*, the cold and the hot, the wind and the rain. With it, one sets in order all the tones and studies their highs and lows (range or register). As long as it is not the sounds of the

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[191] Referring to a chapter in the Book of Rites (*Liji* 禮記), the five sounds, six pitches, and twelve pitch pipes are part of an entire system of revolutionary elements that make up the cosmos.

[192] This is presented in the tables A and B in the analysis section below. Each tone that acts as *gong* had the Seven-Note scale generated from it, and it seems that it also had two additional modes, constructed from the second and fourth pitches respectively.

[193] The "Circulation of Rites" (*liyun* 禮運) is a chapter in the *Book of Rites*, a compilation of ceremonial usages from the Zhou dynasty to the Han dynasty. Its date of compilation is debated amongst scholars, but it may have been compiled in the Western Han. For more details see Loewe, *Early Chinese Texts*, 293.

grass and trees,[194] there is none that cannot harmonize with it. When in the 'Book of Yu'[195] it is said: "the modes harmonize sounds," it refers to this.

房又曰：「竹聲不可以度調，故作準以定數。準之狀如瑟，長丈而十三弦，隱閒九尺，以應黃鍾之律九寸；中央一弦，下有畫分寸，以為六十律清濁之節。」房言律詳於歆所奏，其術施行於史官，候部用之。文多不悉載。故總其本要，以續前志。

Fang also said: "The sound of the bamboo [pipe] cannot be systematically tuned. Therefore [I] made the *zhun*[196] [in order to make] accurate calculations. The shape of the *zhun* resembles a *se*. Its length is one *zhang*[197] and it has thirteen strings. The hollowed (?) space is nine *chi*, corresponding to the nine *cun* of Yellow Bell. A single string [is stretched] in the center, and below it there are markings of the divisions in *cun*, which are used to correctly articulate the degree of the clear and turbid pitches." Fang spoke of the tuning standards in more detail than [Liu] Xin's memorial. His skill was applied by the Astrological officials.[198]

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[194] Likely referring to non-pitch specific sounds.

[195] This is a reference to the chapter titled "the Canon of Shun 舜典" in the *Book of Documents* 尚書. The reference quotes a mysterious segment, dedicated to the mythological musician Kui 夔, who was appointed by the equally mythological emperor Shun to be in charge of musical performance 樂.

[196] *Zhun* 準 can be translated as Equalizer or Standardizer.

[197] The length of the *zhun* would have been 231 cm altogether, and the concealed space would have taken up around 207.9 cm. According to the *zhongguo wulixue shi daxi: Jiliang shi* a *chi* in the Western Han measured around 23.1 cm (a few other measure rulers excavated between 1974-1994 also range up to 23.8 cm). A *cun* in the Western Han is set at around 2.3 cm (with variants as well). Thus one *Zhang*, which is ten *chi* would have been 231cm. See Qiu Guangming 丘光明, *Jiliang Shi = A History of Metrology* 計量史, Zhongguo Wulixue Shi Daxi 中國物理學史大系 (Changsha: *Hunan jiaoyu chubanshe* 湖南教育出版社, 2002), 226–29.

[198] Depending on the context in which it appears, the term 史 *shi* had several meanings in the Qin and Han periods, usually having to do with some kind of "recording of events," which seems to have originated in a religious, ritualistic context (Martin Kern, "The Performance of Writing in Western Zhou China," in *The Poetics of Grammar and the Metaphysics of Sound and Sign*, ed. S. La Porta and D. shulman, Jerusalem Studies in Religion and Culture 6 (Leiden ; Boston: Brill, 2011), 121.). Its translations include "scribe," "clerk," "ritualist, "and "historiographer," among others. (Ibid., 115-116) When considering the duties described in this passage, and their performance in the Department of Astrological Observers (*houbu* 候部), *shi* is best translated as astrologer. (Sivin, *Granting the Seasons*, 35–36.). For a discussion on the origins and early uses of this term in archeological and received sources see Martin Kern, "The Performance of Writing in Western Zhou China," 115.-141.

The department of Astrological Observers used it. Most writings do not record it in detail. Therefore, [I] summarize his main ideas, to supplement the earlier records.

律術曰：陽以圓為形，其性動。陰以方為節，其性靜。動者數三，靜者數二。以陽生陰， 倍之；以陰生陽，四之：皆三而一。

The *Technique of the Tuning Standards* says: "*yang* takes round as its shape. Its nature is dynamic. *Yin* takes square as its shape. Its nature is quiescent. Those that are dynamic number three; those that are quiescent number two. For *yang* to generate *yin*, double it; For *yin* to generate *yang*, quadruple it: for all of these, divide by three.

陽生陰曰下生，陰生陽曰上生。上生不得過黃鍾之（清）濁， 下生不得及黃鍾之（數實）〔清〕。皆參天兩地，圓蓋方覆，六耦承奇之道也。黃鍾，律呂之首，而生十一律者也。其相生也，皆三分而損益之。是故十二律之，得十七萬七千一百四十七，是為黃鍾之實。

When *yang* produces *yin* it is called downward generation. When *yin* produces *yang* it is called upward generation. Upwards generation cannot surpass the most clear and most turbid *Huangzhong*.[199] Downward generation cannot go past the calculation of the numerator[200]of *Huangzhong*.[201] All of these are a matter of odds and evens, round Heaven and square earth; this is the way of six even numbers that bear odd numbers. *Huangzhong* is the fundamental [tone] of the tuning standards, from which eleven pitches are produced. This mutual

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[199] Probably referring to the highest and lowest tones, which may have been predetermined.

[200] Sivin, *Granting the Seasons*, 63.

[201] This passage is difficult, and I have not yet settled on a clear interpretation for it. It would suggest keeping all the pitches within the range of a single octave. Cheng Yingshi argues that Jing Fang's ideas regarding *yin*/*yang* upward and downward pitch generation and his restrictions of range are utterly dismantled by his calculations. This, he claims, is simply part of an intro, and does not mean that his *Changes* interpretation has anything to do with his musical accomplishments (Chen Yingshi 陳應時, "Jing Fang Liushi Lu_ Zai Ban 京房六十律_再辦," *Huangzhong* (*Wuhan Music Conservatory*)《黃钟(武汉音乐学院学报)》 3 (2009): 99–100.)

production in every case divided by three, and one either increases or decreases [from] that. Consequently, the twelve tuning standards yield 177,147, the numerator of the Yellow Bell.[202]

又以二乘而三約之，是為下生林鍾之實。又以四乘而三約之，是為上生太蔟之實。推此上下，以定六十律之實。以九三之，（數）〔得〕萬九千六百八十三為法。

If you double it once more, and divide it by three, this produces downwards the numerator of *Linzhong*. If you multiply it by four and divide it by three, this produces upward the numerator *Taicou*. One iterates this way upward and downward, to order the numerators of all sixty tones. Multiplying three [by itself] nine times, the number obtained is 19,683.

〔於〕律為寸，於準為尺。不盈者十之，所得為分。又不盈十之，所得為小分。以其餘正其強弱。

On the pitch pipes [dimensions are measured] in *cun*, and on the *zhun* [they are measured] in *chi*. For those that are not integral, multiply them by ten and the result is tenths. When multiplying by ten is insufficient, the result is the hundredths. If there is [still] a remainder, standardize [that is, reduce] it to strong and weak parts.[203]

黃鍾，十七萬七千一百四十七。下生林鍾。　黃鍾為宮，太蔟商，林鍾徵。一日。律，九寸。　準，九尺。

Yellow Bell. 177,147. Downward it produces Forest Bell. Yellow Bell is *gong*, Grand Manifold is *shang*. Forest Bell is *zhi*. Day one. Pitch pipe: 9 *cun*. *Zhun:* 9 *Chi*.

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[202] The number for *Huangzhong* was created so that the first twelve tuning standards could be attained without remainders. The number 177,147 is the initial Large Number. It is 3 to the eleventh power (3^{11}). The number 3 had great cosmological significance, and was considered the "quintessential number of the *yang* force." In this process of pitch generation, the largest number must be divisible by 3 twelve times to avoid fractions and remain with integers. Cullen, *The Foundations of Celestial Reckoning*, ??. This worked with a division to sixty pitches as well.

[203] See Granting the Seasons, pp. 85-86.

色育，十七萬六千七百七十六。下生謙待。色育為宮，未知商，謙待徵。六日。律，八寸九分小分八微強。　準，八尺九寸萬五千九百七十三。

Seyu. 176,716. Downward it produces *Qiandai*. *Seyu* is *gong*, *Weizhi* is *shang*, *qiandai* is *zhi*. Six days. Pitch pipe: slightly more than 8.87 *cun*. *Zhun*: 8.8 *chi* and 15,516 (numerator).[204]

執始，十七萬四千七百六十二。下生去滅。　執始為宮，時息商，去滅徵。六日。律，八寸八分小分七大強。準，八尺八寸萬五千五百一十六。

Zhishi. 174,762. Downward it produces *Qumie*. *Zhishi* is *gong*, *Shixi* is *shang*, *Qumie* is *zhi*. Six days. Pitch pipe: slightly more than 8.87…

[All calculations are presented clearly in tables A and B below.]

截管為律，吹以考聲，列以物氣，道之本也。術家以其聲微而體難知，其份數不明，　故作準以代之。準之聲，明暢易達，分寸又粗。然弦以緩急清濁，非管無以正也。均其中弦，令與黃鍾相得，案晝以求諸律，無不如數而應者矣。

One cuts tubes to create pitch-pipes; studies the sounds by blowing [into the pipes]; and arranges [them] according to the *qi* of things.[205] This is the foundation of the Way. Because it was difficult for technicians to understand the subtleties of [the pipe's] sounds, it was hard to know what [relationships] they embodied, and their divisions were unclear, I made the *zhun* to use in its stead. The sound of the *zhun* is clear, smooth and easily obtained. Its divisions of *cun* [that is, the *chi* that correspond to *cun* on the pitch-pipe] are rougher (?)[206]. That being so, by the release or tightening of the strings [the pitch becomes] higher or lower. There is no pipe that cannot be correctly [tuned] with it. By equal division of its middle [that

[204] The denominator is 177,147 (3^{11})

[205] [12 divisions of the tropical year]. Sivin, *Granting the Seasons*, 79–81.

[206] This is still unclear, since there is no reason for Jing Fang to want "rough" divisions. Perhaps they are physically raised to facilitate usage, like bridges on a *se*.

is, single] string, we make it correspond to Yellow bell and use the markings to discover the various pitches. Nothing will fail to correspond to the calculated [tones].

音聲精微，綜之者解。元和元年，待詔候鐘律殷彤上言：「官無曉六十律以準調音者。 故待詔嚴崇具以準法教子男宣，宣通習。願召宣補學官，主調樂器。」

Tones and sounds are profound and subtle. If they are gathered together, they can be explicated. In 84 C.E.[207], the Expectant Observer of Tuning Standards[208] Yin Rong, petitioned the emperor, saying: "None of the officials knows how to tune the sixty tones using the *zhun*. The Expectant Official Yan Chong taught the method of the *zhun* to his son Nan Xuan, and Xuan learned it thoroughly. I wish to summon Xuan to fill a vacancy among the educational officials, and put him in charge of tuning the musical instruments."

詔曰：「崇子學審曉律，別其族，協其聲者，審試。不得依托父學，以聾為聰。聲微妙，獨非莫知，獨是莫曉。以律錯吹. 能知. 命十二律不失一，方為能傳崇學耳。」

The imperial edict said: "Examine Chong's son's erudition by his knowledge of the tuning standards. Separate him from his father, and test and examine his coordination of these sounds. If he does not succeed based on his father's teachings, then his deafness is measure to his wisdom (keenness of hearing).[209] Sound is subtle and delicate: even a single error

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[207] The reign of emperor Zhang of the Eastern Han 章帝 (r.75-88 C.E.).

[208] Hucker translates this title as "Observer of the Bell-like Pitch pipes." His intention may have been that the pitch pipes were used as tuning standards in the same way bells were (possibly) used in earlier periods. Certainly, he is also trying to keep a direct translation. These expectant Officials, together with the Observer of the Air Columns, or Watcher of the Ethers (*Hou Qi* 候氣) were part of the Imperial Observatory (Ling Tai 靈臺). See Hucker 1985, 225.

[209] Meaning his mistakes in accurately recognizing pitches will attest to his lack of true abilities. This combination also appears in the short song (*xiao ge* 小歌) at the end last of the chapter "Fu" 賦 of the *Xunzi*, which refers to the promotion of unworthy men and flawed values. Liang Qixiong 梁啟雄, *Xunzi jianshi* 荀子簡釋, *Beijing di 4 ci yin shua*, *Xinbian zhuzi jicheng xubian* (Beijing: Zhonghua shuju 中華書局, 1983), 363.

proves one is not knowledgeable, while only a single success proves one is not bright. If he can detect (the blowing of) an erroneous mode, and if he iterates the twelve pitches without missing a single one, then he has the ability to pass on Chong's studies."

太史丞弘試十二律，其二中，其四不中， 其六不知何律，宣遂罷。自此律家莫能為準施弦，候部莫知復見。

Deputy Astrologer Hong examined [him] on the twelve pitch pipes: he was right on target in two pitches, but missed four, and could not identify six of them. Xuan's candidacy for appointment was terminated. From this time on, none of the masters of the pitch pipes was able to adjust the strings on the *zhun*. Among the Observers, none was able to set it up ever again.

熹平六年，東觀召典律者太子舍人張光等問準意。光等不知，歸閱舊藏，乃得其器，形制如房書，猶不能定其弦緩急，音不可書以時人，

In 177 C.E., those in charge of the *lü* convened at the Eastern Hall. The palace secretary to the prince, Zhang Guang, and others inquired about the meaning of the *zhun*. Because Guang and the others did not know, they returned and reviewed the old archives. Thereupon they found this instrument. Although the design resembled that in Jing Fang's book, they were unable to determine how tightly to adjust the strings or mark the tones.

知之者欲教而無從，心達者體知而無師，故史官能辨清濁者遂絕。其可以相傳者，唯大撱常數及候氣而已。

Those who knew it desired to teach it, but had no followers, while those with the mental penetration to learn it by heart had no teachers. Thus, the Astrological Officials who could distinguish the clear and turbid pitches died out. This technique could have been passed

down only by elaborate calculations of the regular numerical relationships and the practice of Observing *Qi*.[210]

夫五音生於陰陽，分為十二律，轉生六十，皆所以紀，效物類也。天效以景，地效以響，即律也。陰陽和則景至，律氣應則灰除。

This birth of the five tones out of the *yin* and the *yang*, their division into the twelve *lü* and transformation into sixty, all are ways to record [the divination of *qi*] and model the classification of things.[211] Heaven is modeled by the shadow, and Earth is modeled by the echo.[212] These are the pitch pipes. When the *yin* and the *yang* are harmonized, the shadow reaches [a certain length]. When the pitch pipes and the *qi* correspond, the ashes fly off.

是故天子常以日冬夏至御前殿，合八能之士，陳八音，聽樂均，度晷景，候鐘律，權土灰，(放)[效]陰陽。

Therefore, the Son of Heaven always arrived at the front hall at the summer and winter solstices. Those scholars who combine all Eight Capacities[213] explained the Eight Timbres;

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[210] The solar year was made up of twelve equal divisions called *qi*, which were counterparts of the months, and indicated seasonal changes. Each *qi* period could be further divided in two halves, called nodal *qi* (*jieqi* 節氣) (Sivin, *Granting the Seasons*, 79–80.). The practice of Observing *Q*i, attempted to predict the period of transitions between *qi* periods. A more detailed description is given in the following paragraph.

[211] Following annotation that replaces 斗 with 卦. 卦氣 is a type of *yijing* divination. (look this up)

[212] The distinction between shadow and echo reflects the combination of sight and sound (Goldin's note). These tie into the sagacious sensorial qualities of "keenness of hearing and sight" (*congming* 聰明).

[213] These were eight scholars (*shi* 士) who oversaw the correspondence between tuning and the solar calendar. They made sure that the instruments were tuned according to the correct standards, and as seen from the context of Jing Fang's theory, they not only measured the shadow of the gnomon, but made sure the transition from the use of one scale (and its modes) to the next was carried out at the right time. Their duties are reiterated in the *HHS* "Treatise on Ritual and Etiquette (2)" ("Liyi" 禮儀中), where the ruler summoned them three days prior to the solstices, the times when the gnomon shadow was at its longest or shortest, to assess the accuracy of the musical and astronomical instruments. The two solstices were thought to indicate the extreme states of *qi* and the commencement of nature's seasonal changes. Some of the duties of the scholars of the Eight Capacities included examining the tuning measures, listening to the *Huangzhong* scale on the various instruments (mouth organ 竽, bells, se zither, chimes etc.) to determine their accuracy, measuring the shadow of the gnomon, and leveling the water (權水輕重). This was done in

heard the uniformity of music; measured the shadow of the gnomon; timed the tuning (modes of bells); assessed the ashes of the earth, and modeled the *yin* and the *yang*.

冬至陽氣應，則樂均清，景長極，黃鐘通，土灰輕而衡仰。夏至陰氣應，則樂均濁，景短極，蕤賓通，土灰重而衡低。進退於先後五日之中，八能各以候狀聞，太史封上。效則和，否則佔。

On the winter solstice, the *yang qi* responds, then the pitch is high, the shadow is at its longest, Yellow Bell passes through, and the weight of the earthly ashes are light. In the summer solstice the *yin qi* responds, then the pitch is low, the shadow is at its shortest, Luxuriant Guest passes through, and the earthly ashes are heavy. This takes place within the span of more or less five days. Each of those with the Eight Capacities reports by observing the conditions. The grand Astrologer seals it and petitions the emperor. If it corresponds, then there is harmony; if it does not, then [one should] divine.

候氣之法，為室三重，戶閉，塗壨必遍，密布緹縵。室中以木為案，每律各一，內庳外高，從其方位，加律其上，以葭莩灰抑其內端，案歷而候之。氣至者灰去。

For the method of the Observing *Qi*, use a thrice encircled room with its doors shut, and cover any cracks all the way around with a heavy orange colored curtain. In the center of the room, place a wooden rectangular stand and place each and every pitch pipe so that a short section is buried [in the ground] and the long section sticks out [of the ground]. Following this position, fill the pitch pipes evenly with ash from the membrane of a young reed. Observe it according to the calendar. When the *qi* arrives, the ash departs.

其為氣所動者其灰散，人及風所動者其灰聚。殿中候，用玉律十二。惟二至乃候靈臺，用竹律六十。候日如其歷。

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preparation for a grand solstitial ceremony. *Houhan Shu* 後漢書, 2nd ed. (Beijing: Zhonghua Shuju 中華書局, 1973), 3125–26.

When it is moved by *qi,* the ashes scatter. When it is moved by man or wind, the ashes gather. The Palace Administrator observes by using twelve jade pitch pipes. Observation from the observatory is conducted only on the two solstices. For these, use sixty bamboo pitch pipes. Observe whether the day corresponds to the calendar.

Section Three: Analysis

The introduction opens with a reference to the *Zuozhuan* about numbers (*shu* 數) which readers of the time would have perceived immediately. The parallel treatise in the *Han shu* LLZ quotes the entire sentence, while the *HHS* opening presupposed readers' familiarity with this citation and with the cosmological theory of counterparts and numbers (*xiang* 象 and *shu* 數) it represents, particularly in relation to the *Changes*. Although *Shu* translates as "Numbers," its meaning encompassed many forms of repetitive events, especially those of astrological, astronomical nature, which were interpreted through divination, mathematics, and numerology. Nathan Sivin translates *shu* as "Regularities,"[214] and so conveys how this term was perceived, even during the Han. Ho Peng Yoke also argues that numbers (*shu* 數) were thought to "govern the entire operation of nature," and that no translation would accurately describe this term, since "numbers" is only one of its meanings.[215] Jing Fang's acoustic theory, his practice of *Observing Qi*, his divination according to natural phenomena, and his interpretation of the *Changes* are all connected through the concept of *shu*.

The passage continues with the creation of calculating tools and their combination with measuring tools to create systems of measurements. The use and advancement of these measuring systems were driven by the more abstract search to sum up the "hidden realities and subtle transformations" (*yin zhi qing, jingwei zhi bian* 隱之情，精微之變) around us, which relate to the three main cosmological forces: *yin*, *yang*, and *qi*. According to these

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[214] Sivin, *Granting the Seasons*, 372.

[215] Peng Yoke Ho, *Chinese Mathematical Astrology: Reaching Out to the Stars*, Needham Research Institute Series (London ; New York: RoutledgeCurzon, 2003), 6.

general principles, the up and down tone-generation system is analogous to the correspondence between the forces of *yin* and *yang*, especially apparent in the interpretation of hexagram lines from the *Book of Changes*.[216] The chapter continues with three major figures who were influential in the history of tuning measures: Marquess of Beiping Zhang Cang, Emperor Wu, and Liu Xin. Then, it proceeds to discuss Jing Fang's background and theory.

Jing Fang's cosmo-musical theory

The conceptualization of tuning standards as part of the *yin yang* cosmological scheme is based on the shared idea of mutual generation. In the same way that *yang* produces *yin* and vice versa, pitches also mutually generate one another. Moreover, the twelve tuning standards (*shier lü* 十二律) were sub-divided into two sets. Six pitches were categorized as *yang* pitches also called *lü* 律, and six were categorized as *yin* pitches and called *lü* 呂. A *yang* category pitch generated a *yin* category pitch, which further generated a *yang* category pitch. For example: *Huangzhong*, a *yang* pitch from the *lü* 律 category, mathematically generates *Linzhong*, a *yin* pitch from the *lü* 呂 category. *Linzhong*, in turn, generates *Taicou*, a *yang* tone from the *lü* 律 category. In this way, the first twelve tuning standards fluctuate back and forth in the *yin-yang* cycle.

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[216] The lines of a hexagram are counted from bottom to top. A solid line is treated as *yang* and is also represented by the number 9. A broken line is treated as *yin* and also represented by the number 6 (Edward L. Shaughnessy, *Unearthing the Changes: Recently Discovered Manuscripts of the Yi Jing (I Ching) and Related Texts*, Translations from the Asian Classics (New York: Columbia University Press, 2014), 22.-3). Hexagrams can relate to each other through the transformation of *yang* lines to *yin* lines and vice versa. These inversions create pairs of complimentary or opposite hexagrams. Qian 乾䷀ and Kun 坤䷁ are an example of opposite hexagrams, where all the *yang* lines invert and become *yin* lines. (Shaughnessy, 31.).

Their sub-division is detailed in the parallel treatise in the *Han shu*:[217]

Yang lü 陽律	The *lü* Categorize all things according to a system of *qi* 律以統氣類物.	*Huangzhong* 黃鐘，*Taicou* 太族，*Guxi* 姑洗，*Ruibin* 蕤賓，*Yize* 夷則，*Wangshi* 亡射.
Yin lü 陰呂	The *lü* choose *qi* according to the *yang* classification 呂以族陽宣氣.	*Linzhong* 林鐘，*Nanlü* 南呂，*Yingzhong* 應鐘，*Dalü* 大呂，*Jiazhong* 夾鐘，*Zhonglü* 中呂.[218]

These sets are mentioned twice in the treatise. First, in the general introduction to numbers and measures: "Sound is clear or turbid; it corresponds (cosmologically) using the tuning standards 聲有清濁，協以律呂." The second is part of Jing Fang's explication of the general principals behind his tuning theory: "*Huangzhong* is the fundamental [tone] of the tuning standards, from which eleven pitches are generated 黃鍾，律呂之首， 而生十一律者也."

Jing Fang then immediately dives into the basic principles of the system for pitch generation, known as Addition and Subtraction by a Third (*Sanfen Sunyi* 三分損益). His

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[217] Ban Gu 班固, *Han shu* 漢書, 4:958–59.

[218] The additional subdivision of the six *yin lü* into two subsets, three ending in *lü* 呂 and three ending in *zhong* 鍾 (bell), which also alternate. This suggests an earlier categorization system that was somehow unified. However, this only applies to *yin* category pitches (呂). *Yang* category pitches carry a different ending for each pitch name. *Huangzhong* is the only pitch with a familiar *zhong* 鍾 ending. I will not elaborate on this here. Falkenhausen has raised this issue and argues for a diachronic development of pitch nomenclature in the Zhou dynasty. Lothar von Falkenhausen. "On the Early Development of Chinese Musical Theory: The Rise of Pitch-Standards." *Journal of the American Oriental Society* 112, no. 3 (1992): 433–39.

very short introduction to the philosophical ideas that underpin his theory likely lies in its tutorial purpose, to be used by technicians in the department of Astrology, as indicated later in the treatise. These technicians needed little knowledge of the reasons behind the computations. What Martzloff clarifies regarding astronomical texts that were used in "particular domains," applies here as well: "In fact, all this is understandable, if one acknowledges that Chinese astronomical texts which have reached us were intended not for logicians but above all for technicians, mechanically carrying out repetitive, clearly defined tasks"[219]

In this system, one adds or subtracts one third from a given number to produce the subsequent number. The ratio between these two numbers is translated to a musical interval:[220]

> 以上生下，皆三生二，以下生上，皆三生四；阳下生阴，阴上生阳，终于中吕，而十二律毕矣。
>
> [When] up produces down, all threes produce twos. [When] down produces up, then all threes produce fours. When the *yang* goes downward it produces *yin*, and when *yin* goes upward it produces *yang*. When the cycle ends at the *Zhonglü* pitch, the twelve pitches are thus complete! When *Zhonglü* goes upwards it produces [the pitch] *Zhishi*. When *Zhishi* goes downwards it produces [the pitch] *Qumie*. Upwards and downwards they produce each other, culminating at *Nanshi*. The sixty pitches are thus complete!

When we reduce a third from a given number, the subsequent number would equal two thirds of the original value. The ratio between these numbers would be 3:2 (an interval called a fifth). Also, when we take a number and add a third of its value to it, the subsequent number would equal four thirds of the original value. The ratio between these two numbers

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[219] Jean-Claude Martzloff, *A History of Chinese Mathematics*, 2nd ed. (New York: Springer, 2006), 46–47.

[220] See section on the *Sanfen Sunyi* tuning system in appendix section on tuning and instruments.

would be 4:3 (an interval called a fourth). In this way, *yang* pitches produce *yin* pitches and vice versa, creating the numbers and ratios for the first twelve tuning standards, ending on a pitch called *Zhonglü* 中吕. Jing Fang continues the cycle beyond the twelfth pitch, to produce sixty pitches, concluding in a pitch called *Nanshi* (Southern Affairs).

The next section explains how these pitches nestle comfortably in a Seven-Note scale pattern, constructed from the Five-Note pattern prevalent in early Chinese texts, with an addition of two "transformed" notes.[221] The cycle begins on the *Huangzhong* pitch, and is superimposed in ascending order onto the days of the calendar, starting at the winter solstice. Each of the first seven pitches occupies the duration of a single day. The rest of the pitches occupy a varying number of days (between five and eight). The relation between sound, *qi,* and the calendar is made clear at this point. This can be seen clearly in Table B below.

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[221] The notes *Altered gong* 變宫 and *Altered zhi* 變徵 are a continuation of the Five-Note scale. After the fifth note, the nomenclature repeats the cycle, but in order of generation, rather than in ascending order (continuing the fifths/fourths tone production). The first five notes are organized in ascending order, rather than order of generation. This is probably because they are somehow still considered external to the o*rig*inal scale, and so the interval between them (a fourth), is kept. They are not adjacent. Thus, first comes *Altered gong* and a fourth below it is the *Altered zhi*. In Western nomenclature (using untempered, Pythagorean tuning), the Seven-Note scale would be: Do – Re – Mi – Fa# -– Sol – La – Si, where Si and Fa# are the altered notes, respectively. (In order of generation they would be: Do – ↑5th – Sol – ↓4th – Re – ↑5th – La – ↓4th – Mi – ↑5th – Si – ↓4th – Fa#).

The only other example that refers to the continuation of tone generation by the creation of an Altered (*bian* 變) cycle is in the *Huainanzi* "Terrestrial Forms" chapter (*Di Xing Xun* 墜形). This example does not point to a specific scale, but to the technique. Moreover, it provides the entire Altered scale, cycling back to *gong*:

> Altered *gong* generated *zhi.* Altered *zhi* generates *shang*. *Altered shang* generated *yu*. Altered *yu* generates *jue*. Altered *jue* generates *gong*. 變宮生徵，變徵生商，變商生羽，變羽生角，變角生宮.

These cosmo-musical principles are reiterated and elaborated later in the treatise. The way Jing Fang obtains his results is by doubling the number and dividing by it three, or quadrupling the number and dividing it by three. He provides the following examples:

> 1) 以陽生陰， 倍之；以陰生陽，四之：皆三而一。
>
> For *yang* to generate *yin*; double it. For *yin* to generate *yang*; quadruple it. For all of these, divide by three.
>
> 2)又以二乘而三約之，是為下生林鍾之實。又以四乘而三約之，是為上生太蔟之實。
>
> If you double it [*Huangzhong*] once more, and divide it by three, this produces downwards the numerator of *Linzhong*. If you multiply it by four and divide it by three, this produces upward the numerator *Taicou*.

The results are as follows:

Lingzhong: $\frac{177{,}147\times 2}{3} = 118{,}098$ *Taicou*: $\frac{118{,}098\times 4}{3} = 157{,}464$

Additionally, Jing Fang provides his results in pitch pipe and string measurements. Other than acting as proof against mistakes, these measurements clarify that a generation of a larger number (adding a third, resulting in the interval ratio of a fourth), creates a lower pitch, evident by longer pipe and string lengths. In the same way, the generation of a smaller numbers (subtracting a third, resulting in the interval ratio of a fifth), creates a higher pitch, evident by shorter pipe and string lengths. Thus, the measurements show that the system generates falling fourths and rising fifths. This information, with additional explications of Jing Fang's calculations are neatly presented in two tables in the following section.

Tuning Theory: The Jing Fang Comma

In 1979, Ernest G. McClain and Ming Shui Hung published an explanation of Jing Fang's calculations of the sixty tuning standards in English. Their detailed explanations are of great value, but include statements that would not hold today. In general, the article ignores Jing's cosmological motivation, and presents the treatise from a Western perspective. Wang Zichu 王子初[222] as well as Fritz Kuttner[223] before him argued for Jing Fang's indubitable impact on the development of tuning throughout Chinese history, culminating in Zhu Zaiyu's (朱載堉 1536 – 1611) calculation of the equally tempered scale in the Ming dynasty. The detailed work by Chen Yingshi 陳應時, who emphasizes pure musicological issues, is especially notable. Most scholarship on Jing Fang's sixty pitches is Chinese, and much of it is characterized by the notion of "science versus mysticism," which dominated Chinese scholarship in the positivist sixties and seventies, well into the eighties.[224]

What are Jing Fang's sixty modes?

A comma is a problem specifically related to tuning systems. Simply put, it is a gap between two enharmonic tones. Several types of commas have been detected throughout the history of tuning, but I will elaborate only on the comma Jing Fang worked to reduce, known in the

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[222] Wang Zichu 王子初, "*Jing Fang he dade liushi lü* 京房和他的六十律," *Zhongguo Yinyue* 中國音樂 3 (1984): 24–26.

[223] Fritz A. Kuttner, "Prince Chu Tsai-Yü's Life and Work: A Re-Evaluation of His Contribution to Equal Temperament Theory," *Ethnomusicology* 19, no. 2 (May 1975): 163–206.

[224] Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' Jia Ban 為"京房六十律"申辦," *Yiyuan: Yinyue Ban* 藝苑: 音樂版, 1986, 6–13.

west as the Pythagorean comma (in the Western Han, it did not yet have a name). His goal for reducing the comma was to reach the end of the tuning cycle with a pitch as close as possible to the opening pitch. This meant getting as close as possible to the number 177,147 – the number that signifies the *Huangzhong* pitch.[225] The miniscule gap between the original pitch and the closest pitch Jing Fang achieved is known in China as the Jing Fang Comma (*Jing Fang yincha* 京房音差).[226]

I will use simple modern terms to explain how the Pythagorean comma is created. When we discuss musical intervals, we refer to the space between two pitches. This space is expressed in ratios. Modern acousticians translate interval ratios to a logarithmic unit of measure called Cents. An interval of one octave (a ratio of 2:1) equals 1200 Cents. When they are divided equally between the twelve tones, each semitone equals 100 Cents. A scale built with this system is called an equal temperament scale, because the intervals were adjusted so that there is an equal, fixed interval between each adjacent pair of tones, eventually reaching exactly one octave ($100 \times 12 = 1200$).

A different tuning system, called Pythagorean tuning, uses intervals of a fifth (a ratio of 3:2) and octaves to create a scale. This method is closest to the *Sanfen Sunyi* method of early China, which uses fifths, but not octaves.[227] The problem with this type of system is that after calculating twelve consecutive fifths, which create the twelve tones of a chromatic scale, the thirteenth tone slightly overshoots the octave pitch.[228] This is because while an

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[225] See the first two pitches in table A

[226] Wang Zichu 王子初, "*Jing Fang he dade liushi lü* 京房和他的六十律," 24.

[227] See the history and explication of the *Sanfen Sunyi* tuning method in chapter 1.

[228] An example of generating all twelve chromatic tones using fifths: Do-Sol-Re-La-Mi-Si-Fa#-Do#-Sol#-Re#-La#-Mi#

equally tempered fifth equals 700 Cents (100×7 semitones), a Pythagorean fifth is not adjusted, and equals 701.955 Cents (an exact and untempered 3:2 ratio). This extra 1.955 Cents accumulate with the generation of each consecutive fifth. Now, twelve consecutive fifths are supposed to reach the same pitch as that which seven octaves would reach. However, they don't. Viewing this is Cents is easier: Seven octaves equal 8400 Cents (1200×7), while twelve Pythagorean fifths equal 8,423.46 Cents (701.955×12). The nearly two additional cents carry over twelve times, eventually resulting in an audible difference of 23.46 Cents (1.955×12), known in the West as a Pythagorean comma. Below, I will show how Jing Fang dealt with the same problem.

The sixty tuning standards are a solution to the comma problem. They are an attempt to reduce the gap between the first tone, *Huangzhong* and thirteenth, *Zhishi*. Transcribing the ratio between these two tones (177,147: 174,762) into Cents, we reach the comma: 23.46 Cents (which equals a Pythagorean comma).[229] Jing Fang's calculations also teach us that the term *lü* 律 referred to tuning and thus to the effort of standardizing measures. The term means more than tuning standards, and certainly more than pitch pipes. It included the scales (five to seven tone scales built on each of the pitch standards, which in turn acted as fundamental tones), and likely also the modes built on the first, second, and fourth pitches of the scale, which some treat as movable Do system (*Xuan gong* 旋宫), added on top of the tuning standards generation system (*Sanfen Sunyi*).[230]

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[229] In order to do this one inserts the ratio into the following logarithm, which translates ratios into Cents: 1200*log (m/n)/log (2), where m/n is the ratio. Thus: $1200 \log(177{,}147 \div 174{,}762) \div \log(2) = 23.466614531$.

[230] Chen Yingshi 陳應時, "Jing Fang Liushi Lu_ Zai Ban 京房六十律_再辦," 99.

But we may ask: what would happen if he would have gone beyond the 60 tuning-measures, wouldn't he have gotten even closer? Using powers of three would never allow an exact return to the Large Number of *Huangzhong* using the *Sanfen Sunyi* system. Cheng Yingshi argues that Jing Fang realized this and so ceased his calculation when reaching a number that answered his conditions as he saw fit. However, in the Southern Song Jing Fang's experiment was extended to 360 tuning-measures, reducing the Jing Fang comma even further, from 3.63 cents to 1.845 cents.[231]

Synchronization of cycles as motivation to solve the comma problem

None of the early texts that introduce the *Sanfen Sunyi* tuning-measure system went beyond the twelfth pitch generation, and so none raised any problems the system might have, possibly because they didn't see one. Jing Fang's goal to cycle back to *Huangzhong* (177,147), as attested in the translation section of this chapter, was inspired by a quote from the "Circulation of Rites" (*Liyun* 禮運) chapter in the *Book of Rites* (*Liji* 禮記): "The five sounds, six pitches, and twelve pipes circle back to *gong*." He does not explicate this idea further in the treatise, and the connection between his calculation and this quote is not easily seen.

Chen Yingshi argues for a link between the two: Adding six pitches to the 13th pitch produces the second closest Seven-Note scale to the fundamental scale (*zheng lü* 正律) of *Huangzhong* (*Zhishi* is the second closest pitch to *Huangzhong* after *Seyu*. The Seven-Note scale

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[231] Kuttner, "Prince Chu Tsai-Yü's Life and Work: A Re-Evaluation of His Contribution to Equal Temperament Theory," 172. (I'm not sure whether he also used the Seven-Note scale to complete the cycle on the winter solstice. If not, this might be the reason Jing Fang did not continue. See elaboration on this topic in the section on motivation).

would be from *Zhishi* to *Chinei* 遲內).[232] This would have answered only the condition of constructing a scale close to the fundamental [233], but the end of this cycle would not fall on the winter solstice, which is the second condition required in the "Li Yun" quote Jing Fang provided.[234] Jing Fang's intention was not merely "Returning to *Huangzhong*" (還原黃鐘), which would mean solving the problem of circling back to the opening *Huangzhong* pitch, but he also wished to draw out the circle of the mutual progression (還相) from the *gong* note (1st position) to conclude all seven notes of the scale he offered in the opening of his theory exactly at the winter solstice.[235] This, in my view, created a type of conjunction between three cycles: notes, pitches, and the calendar. This mode of thought is very reminiscent of the cyclical astronomical systems in the Qin and Han, which had the same intention.[236]

Kuttner, on the other hand, suggests that Jing Fang selected twelve pitches that reach closely to the Large Numbers of the original twelve pitches (*Huangzhong* to *Zhonglu*).[237] Kuttner may be objectively correct, but he ignores the scale Jing Fang presented in the beginning of the treatise, and does not explain why Jing Fang continued past the 54th tuning-measure to generate six more pitches, reaching the sixtieth pitch, *Nanshi* 南事. Chen Yingshi's argument provides solid musicological and intellectual reasoning behind these last

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[232] This was done in the Song dynasty by Cai Yuanding **蔡元定(**1135－1198) in his book *Lülü Xinshu* 律呂新書

[233] The Pythagorean comma is apparent in the ratio between *Huangzhong* and *Zhishi* (23.46 Cents)

[234] Chen Yingshi 陳應時, "Jing Fang Liushi Lu_ Zai Ban 京房六十律_再辦," 98.

[235] Chen Yingshi 陳應時, "Jing Fang Liushi Lu_ Zai Ban 京房六十律_再辦."

[236] For a detailed introduction to the Triple Concordance and the Quarter Day astronomical systems see Sivin, "Cosmos and Computation," and Cullen, *The Foundations of Celestial Reckoning*.

[237] Kuttner, "Prince Chu Tsai-Yü's Life and Work: A Re-Evaluation of His Contribution to Equal Temperament Theory," 172.

six pitches. *Huangzhong* and *Seyu* provide the example for the reduction of the comma (3.62 Cents instead of 23.46 Cents between *Huangzhong* and *Zhishi*), known as the Jing Fang comma. He continued because was interested in calculating an entire Seven-Note scale that would match as close as possible to the original Seven-Note *Huangzhong* scale (*yun* 均).[238]

The astronomical year was divided into temporal and spatial sectors (Solar Terms, *hou* 候, and Lodges, *xiu* 宿). Each tuning standard and its accompanying two modal transformations[239] occupied a period from one to eight days of the year, and was related to a certain portion of the celestial map of that period. Based on each tone, one can construct a Seven-Note scale, with its accompanying modal transformations. The scale would be *gong* 宮, *shang* 商, *jue* 角, *zhi* 徵, *yu* 羽, *bian gong* 變宮, *bian zhi* 變徵. The intervals in this Seven-Note scale, create the following pattern:

[238] See table B. Each of the numbers in the first Seven-Note scale (*Huangzhong* to *Nanzhong*) is separated from their equivalents in the last seven pitches in the table (*Seyu* to *Nanshi*) by the miniscule Jing Fang comma (3.62 Cents).

[239] We can think of modes in this context as cycling through a chosen scale, each time beginning at a different position. Jing Fang provides us with two possible starting points, the second position, and the fourth position of the scale. For example, a Seven-Note *shang* 商 mode would contain *shang* 商, *jue* 角, *zhi* 徵, *bian gong* 變宮, *yu* 羽, *bian zhi* 變徵. However, we cannot be certain on how these notes were used, or whether all of them were in use. As we can see, only three modes are used for each scale. There is no reason that something may have occurred not happen within the note of the scale.

宮– Pth[240] whole tone (203.91 Cents)[241] – 商– Pth whole tone-角 – Pth whole tone – 變宮 – Pth small semi tone (90.225 Cents)[242] – 徵 – Pth whole tone – 羽 – Pth Whole tone – 變商– Pth large semitone (113.691) – 宮.[243]

The following table organizes the data as it appears in the treatise. I provide Jing Fang's pitch names organized in ascending order.[244] Table B presents the same information, but with the pitches organized according to their order of generation. Organizing the information in a table facilitates bringing into view a few characteristics of the way Jing Fang perceived sound:

1. Jing Fang (and any other early Chinese tuning formulas) present interval ratios and not exact pitches. When constructing these scales, one can begin at any pitch and apply the system. The only indication regarding the actual pitch height is provided by the string and pipe length he added.

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[240] "Pth" stands for "Pythagorean." This means that the tone-generation technique uses sequences of fifths (and fourths), which is different from modern tempered tuning technique. Thus, the intervals are not the same size as those used in equal temperament. In the Pythagorean scale, whole tones are a bit larger and semitones are a bit smaller than their equivalents in the tempered scale. These differences are more easily audible when two or more notes are played simultaneously.

[241] A tempered whole tone interval is 200 Cents. See p. 100 for a note on Cents.

[242] This interval is also known in Pythagorean intervals as Limma. A tempered semitone interval is 100 Cents.

[243] In the chromatic twelve tone scale, a large semitone (113.691 Cents) is also created between E and F (= E#).

[244] In this order, the pitch frequency ascends gradually, as the numbers descend and the pipe and string length shorten. Thus, the first fundamental, *Huangzhong*, which has the number 177,147 is the lowest tone, and the following microtones gradually ascend in extremely small increments. The problem with the numbers, of course, is that they only signify ratios, rather than units of any measure. The Equilizer and Pipe are the only ones that could potentially give us specific knowledge of pitch frequency. What the numbers do show us, is the ratio between the tones. The ratio translates into fourths and fifth intervals.

2. The pitches are presented in the treatise in ascending order. This means that the pitch following *Huangzhong* is not the pitch it generates[245], but the one closest to *Huangzhong* in pitch and thus in Large Numbers, the 54th tone *Seyu* 色育. To minimize confusion, the second column provides the place number of the pitch in the order of generation (*Huangzhong* is the 1st pitch, and *Seyu* is the 54th pitch).[246]
3. Downward generation (下生) produces a higher pitch, while upward generation (上生) produces a lower pitch. This may be instinctively confusing, but it is also evident by the string and pitch pipe lengths (longer length equals lower pitch).
4. Jing Fang constructed a Seven-Note scale on each pitch. Each period (lasting between one and eight days) is characterized by an entire scale, and not by a single pitch. In addition to the scale built on the fundamental, he also provided an option for two additional modes, which would be built on the second (*shang* 商) and fourth (*zhi* 徵) positions of the original scale. This way, each time period is characterized by a scale and two modes.[247]
5. On the 6th generated pitch there is a double upward generation from the 6th (*Yingzhong* 應鐘) to the 7th (*Ruibin* 蕤賓), and from the 7th (*Ruibin*) to the 8th (*Dalü* 大呂). This means the pitches drop by a fourth twice in a row (for example from B down to F# and then down again to C# of the same octave, instead of up a fifth to C# of the following

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[245] The pitch *Huangzhong* generates is *Linzhong* 林種, a fifth above *Huangzhong*.

[246] McClain also adds a column like this in his table.

[247] The modes are not stated, but they can be inferred with quite certainty by the way they are presented. Jing Fang only provides the first, second a fourth pitch names for each scale, and he presented the overall Seven-Note scale earlier in the treatise. Thus, the scale pattern would be initially applied to a fundamental pitch, and then two additional modes can be created by beginning the scale from the second or from the fourth position.

octave). This allows Jing Fang to keep within the range of a single octave. This is highlighted in table B.

6. In Table A, the interval between each pitch and the next is miniscule, and would not be audible to the human ear.

7. If a machine capable of producing these microtones were to play the pitches in Table A one after the other, it would sound like a continuous upscale glide within the interval of a single octave (If it were to play the pitches in Table B it would sound like a continuous upward and downward oscillation between rising fifths and falling fourths, except on *Ruibin*, as shown above.

Table A: Jing Fang's Sixty Tuning Standards Organized in Ascending Order

The first column shows pitch names, followed by their order of generation. Column three indicates their Large Numbers. These numbers are used to determine the ratio between each pair of pitches, and allows us to examine the results of Jing Fang reducing the size of the comma. The fourth and fifth columns indicate the following pitch in the order of generation (these pitches would be either a fifth above or a fourth below the preceding pitch). The sixth column gives three options for generating modes based on the first, second, and fourth positions of the scale. The seventh column indicates the duration of each pitch and its accompanying modes in number of days. The eighth column expresses the pitch in string length, while the ninth column shows its length in pitch pipes. Lastly, I added a column that translates the ratios into Cents, to facilitate understanding of the intervals in modern terms.

Table A Key:

A	Tones (order by Text)	F	Mode Options
B	Order of Generation	G	Number of Days
C	Numbers[248]	H	律 on a pitch pipe (in *cun*)[249]
D	上生 Generate Upward	I	準 *zhun* (in *chi*)
E	下生 Generate Downward	J	Cents

[248] Reminder: We measure the ratio between two numbers to determine their interval. That interval can then be converted into Cents. The modern tempered octave contains twelve tones, and is divided into 1200 Cents. The interval between each two adjacent tones, also known as a semitone, equals 100 Cents. Knowing this, we can appreciate the miniscule interval between *Huangzhong* and *Seyu*, which stands at 3.6295 Cents.

[249] I do not provide the length measurments for the pitch pipe and the *zhun* in full. McClain and Ming Shui Huang provide them with additional commentary on the various remainders. The measurements show that the pipe lengths are very short, and therefore their remainders are not functional. The *zhun* measurements are larger, thus easier to apply, asserting the purpose of its making.

A	B	C	D	E	F	G	H	I	J
黃鍾 *Huangzhong*	1	177,147		林鍾 *Linzhong*	黃鍾為宮，太蔟商，林鍾徵 Huangzhong is gong (1), Taicu is shang (2), Linzhong is zhi (4)	1 WS	9	9	
色育 *Seyu*	54	176,776		謙待 *Qiandai*	色育為宮，未知商，謙待徵	6			**3.6295**
執始 *Zhishi*	13	174,762		去滅 *Qumie*	執始為宮，時息商，去滅徵	6			19.8371
丙盛 *Bingcheng*	25	172,410		安度 *Andu*	丙盛為宮，屈齊商，安度徵	6			23.4577
分動 *Fendong*	37	170,089		歸嘉 *Guijia*	分動為宮，隨期商，歸嘉徵	6			23.4643
質末 *Zhimo*	49	167,800		否與 *Fouyu*	質末為宮，形晉商，否與徵	6			23.4566
大呂 *Dalü*	8	165,888		夷則 *Yize*	大呂為宮，夾鍾商，夷則徵	8			19.8398
分否 *Fenfou*	20	163,654		解形 *Jiexing*	分否為宮，開時商，解形徵	8			23.4728
凌陰 *Lingyin*	32	161,452		去南 *Qunan*	凌陰為宮，族嘉商，去南徵	8			23.4523
少出 *Shaochu*	44	159,280		分積 *Fenji*	少出為宮，爭南商，分積徵	6			23.4482
太蔟 *Taicu*	3	157,464		南呂 *Nanlü*	太蔟為宮，姑洗商，南呂徵	1	8	8	19.8517
未知 *Weizhi*	56	157,134		白呂 *Bailü*	未知為宮，南授商，白呂徵	6			**3.632**
時息 *Shixi*	15	155,344		結躬 *Jiegong*	時息為宮，變虞商，結躬徵	6			

A	B	C	D	E	F	G	H	I	J
屈齊 *Quqi*	27	153,253		歸期 *Guiqi*	屈齊為宮，路時商，歸期徵	6			
隨期 *Suiqi*	39	151,190		未卯 *Wiemao*	隨期為宮，形始商，未卯徵	6			
形晉 *Xingjin*	51	149,156		夷汗 *Yihan*	形晉為宮，依行商，夷汗徵	6			
夾鍾 *Jiazhong*	10	147,456		無射 *Wuyi*	夾鍾為宮，中呂商，無射徵	6			
開時 *Kaishi*	22	145,470		閉掩 *Biyan*	開時為宮，南中商，閉掩徵	8			
族嘉 *Zujia*	34	143,513		鄰齊 *Linqi*	族嘉為宮，內負商，鄰齊徵	8			
爭南 *Zhengnan*	46	141,582		期保 Qibao	爭南為宮，物應商，期保徵	8			
姑洗 *Guxi*	5	139,968		應鍾 *Yingzhong*	姑洗為宮，蕤賓商，應鍾徵	1			
南授 *Nanshou*	58	139,674		分烏 *Fenwu*	南授為宮，南事商，分烏徵	6			**3.6402**
變虞 *Bianyu*	17	138,084		遲內 *Chinei*	變虞為宮，盛變商，遲內徵	6			
路時 *Lushi*	29	136,225		未育 *Weiyu*	路時為宮，離宮商，未育徵	6			
形始 *Xingshi*	41	134,392		遲時 *Chishi*	形始為宮，制時商，遲時徵	5			
依行 *Yixing*	53	132,582	色育 *Siyu*		依行為宮，謙待商，色育徵	7			
中呂 *Zhonglü*	12	131,072	執始 *Zhishi*		中呂為宮，去滅商，執始徵	8			
南中 *Nanzhong*	24	129,308	丙盛 *Bingcheng*		南中為宮，安度商，丙盛徵	7			

A	B	C	D	E	F	G	H	I	J
內負 *Neifu*	36	127,567	分動 *Fendong*		內負為宮，歸嘉商，分動徵	8			
物應 *Wuying*	48	125,850	質末 *Zhimo*		物應為宮，否與商，質末徵	7			
蕤賓 *Ruibin*	7	124,416	大呂 *Dalü*		蕤賓為宮，夷則商，大呂徵	1			
南事 *Nanshi*	60	124,154	None	None	南事窮，無商、徵，不為宮	7			**3.6495**
盛變 *Chengbian*	19	122,741	分否 *Fenfou*		盛變為宮，解形商，分否徵	7			
離宮 *Ligong*	31	121,089	凌陰 *Lingyin*		離宮為宮，去南商，凌陰徵	7			
制時 Zhishi	43	119,460	少出 *Shaochu*		制時為宮，分積商，少出徵	8			
林鍾 *Linzhong*	2	118,098	太蔟 *Taicou*		林鍾為宮，南呂商，太蔟徵	1	6	6	
謙待 *Qiandai*	55	117,851	未知 *Weizhi*		謙待為宮，白呂商，未知徵	5			**3.6246**
去滅 *Qumie*	14	116,508	時息 *Shixi*		去滅為宮，結躬商，時息徵	7			
安度 *Andu*	26	114,940	屈齊 *Quqi*		安度為宮，歸期商，屈齊徵	6			
歸嘉 *Guijia*	38	113,393	隨期 *Suiqi*		歸嘉為宮，未卯商，隨期徵	6			
否與 *Fouyu*	50	111,867	形晉 *Xingjin*		否與為宮，夷汗商，形晉徵	5			
夷則 *Yize*	9	110,592	夾鍾 *Jiazhong*		夷則為宮，無射商，夾鍾徵	8			
解形 *Jiexing*	21	109,103	開時 *Kaishi*		解形為宮，閉掩商，開時徵	8			

A	B	C	D	E	F	G	H	I	J
去南 *Qunan*	33	107,635	族嘉 *Zujia*		去南為宮，鄰齊商，族嘉徵	8			
分積 *Fenji*	45	106,187	爭南 *Zhengnan*		分積為宮，期保商，爭南徵	7			
南呂 *Nangong*	4	104,976	姑洗 *Guxi*		南呂為宮，應鍾商，姑洗徵	1			
白呂 *Bailü*	57	104,756	南授 *Nanshou*		白呂為宮，分烏商，南授徵	5			**3.632**
結躬 *Jiegong*	16	103,563	變虞 *Bianyu*		結躬為宮，遲內商，變虞徵	6			
歸期 *Guiqi*	28	102,169	路時 *Lushi*		歸期為宮，未育商，路時徵	6			
未卯 *Weimao*	40	100,794	形始 *Xingshi*		未卯為宮，遲時商，形始徵	6			
夷汗 *Yihan*	52	99,437	依行 *Yixing*		夷汗為宮，色育商，依行徵	7			
無射 *Wushe*	11	98,304	中呂 *Zhonglü*		無射為宮，執始商，中呂徵	8			
閉掩 *Biyan*	23	96,980	南中 *Nanzhong*		閉掩為宮，丙盛商，南中徵	8			
鄰齊 *Linqi*	35	95,675	內負 *Neifu*		鄰齊為宮，分動商，內負徵	7			
期保 *Qibao*	47	94,388	物應 *Wuying*		期保為宮，質末商，物應徵	8			
應鍾 *Yingzhong*	6	93,312	蕤賓 *Ruibin*		應鍾為宮，大呂商，蕤賓徵	1			
分烏 *Fenwu*	59	93,116	南事 *Nanshi*		分烏窮次，無徵，不為宮	7			**3.6402**
遲內 *Chinei*	18	92,056	盛變 *Chengbian*		遲內為宮，分否商，盛變徵	8			

A	B	C	D	E	F	G	H	I	J
未育 *Weiyu*	30	90,817	離宮 *Ligong*		未育為宮，凌陰商，離宮徵	8			
遲時 *Chishi*	42	89,595	制時 *Zhishi*		遲時為宮，少出商，制時徵	6			
					Total:	366			

Table B: Jing Fang's Sixty Tuning Standards Organized by Order of Generation

In this table pitches are organized according to their order of their generation. This means each pitch is a (Pythagorean) fifth above or a fourth below the preceding pitch. The first seven tuning standards produce all the pitches included in the seven-tone scale Jing Fang mentions (albeit not in their final and ascending order, but in the order of their generation).

This table allows a clear view of the first and last seven tones. The comma (the gap between the first tone and the thirteenth) is reduced to satisfaction on the fifty fourth pitch, *Seyu* 色育 176,776 (which is the second pitch in Table A). Its number is closest to the outset point of *Huangzhong* 177,147 and audibly indistinguishable from it. Jing Fang continues the cycle onto tone 60 in order to complete the entire gamut of a seven-tone scale starting at *Seyu*. This would match the seven-tone scale starting at *Huangzhong*.[250] One can also notice that the ratio between the *Huangzhong* pitch and the thirteenth pitch *zhishi* is 177,147/ 174,762, which is an interval of 23.46 Cents. This interval is the gap Jing Fang is reducing, known as the Pythagorean comma.

[250] Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' Jia Ban 為"京房六十律"申辦."

Key for Table B:

A	Tuning Standard		F	Mode Options
B	Order of Generation		G	Number of Days
C	Large Numbers[251]		H	X
D	上生 Generate Upward		I	X
E	下生 Generate Downward		J	Cents (showing intervals of fourths and fifths)

[251] Reminder: We measure the ratio between two numbers to determine their interval. That interval can then be converted into Cents. The modern tempered octave contains twelve tones, and is divided into 1200 Cents. The interval between each two adjacent tones, also known as a semitone, equals 100 Cents. Knowing this, we can appreciate the miniscule interval between *Huangzhong* and *Seyu*, which stands at 3.6295 Cents.

A	B	C	D	E	F	G	J
黃鍾 *Huangzhong* (1 宮) [Do]	1	177,147		林鍾	黃鍾為宮，太蔟商，林鍾徵 *Huangzhong* is *gong* (1), *Taicu* is *shang* (2), *Linzhong* is *zhi* (4)	1 WS[252]	
林鍾 *Linzhong* (4 徵) [Sol]	2	118,098	太蔟		林鍾為宮，南呂商，太蔟徵	1	701.955 (fifth)
太蔟 *Taicou* (2 商) [Re]	3	157,464		南呂	太蔟為宮，姑洗商，南呂徵	1	498.045 (fourth)
南呂 *Nanlü* (5 羽) [La]	4	104,976	姑洗		南呂為宮，應鍾商，姑洗徵	1	701.955 (fifth)

[252] Each of the initial seven tuning standards and their accompanying modal options occupy a single day.

A	B	C	D	E	F	G	J
姑洗 *Guxi* (3 角) [Mi]	5	139,968		應鍾	姑洗為宮，蕤賓商，應鍾徵	1	498.045 (fourth)
應鍾 *Yingzhong* (7 變宮) [Si]	6	93,312	蕤賓[253]		應鍾為宮，大呂商，蕤賓徵	1	Etc.
蕤賓 *Ruibin* (6 變徵) [Fa#]	7	124,416	大呂		蕤賓為宮，夷則商，大呂徵	1	
大呂 *Dalü*	8	165,888		夷則	大呂為宮，夾鍾商，夷則徵	8	
夷則 *Yize*	9	110,592	夾鍾		夷則為宮，無射商，夾鍾徵	8	
夾鍾 *Jiazhong*	10	147,456		無射	夾鍾為宮，中呂商，無射徵	6	

[253] Double upward generation

A	B	C	D	E	F	G	J
無射 *Wuyi*	11	98,304	中呂		無射為宮，執始商，中呂徵	8	
中呂 *Zhonglü*	12[254]	131,072	執始		中呂為宮，去滅商，執始徵	8	
執始 *Zhishi*	13[255]	174,762		去滅	執始為宮，時息商，去滅徵	6	
去滅 *Qumie*	14	116,508	時息		去滅為宮，結躬商，時息徵	7	
時息 *Shixi*	15	155,344		結躬	時息為宮，變虞商，結躬徵	6	
結躬 *Jiegong*	16	103,563	變虞		結躬為宮，遲內商，變虞徵	6	
變虞 *Bianyu*	17	138,084		遲內	變虞為宮，盛變商，遲內徵	6	
遲內 *Chinei*	18	92,056	盛變		遲內為宮，分否商，盛變徵	8	

[254] End of the generation of the twelve chromatic tuning standards (*shier lü* 十二律)

[255] *Zhishi,* the 13th tuning-standard, creates a Pythagorean Comma in relation to *Huangzhong*.

A	B	C	D	E	F	G	J
盛變 *Chengbian*	19	122,741	分否		盛變為宮，解形商，分否徵	7	
分否 *Fenfou*	20	163,654		解形	分否為宮，開時商，解形徵	8	
解形 *Jiexing*	21	109,103	開時		解形為宮，閉掩商，開時徵	8	
開時 *Kaishi*	22	145,470		閉掩	開時為宮，南中商，閉掩徵	8	
閉掩 *Biyan*	23	96,980	南中		閉掩為宮，丙盛商，南中徵	8	
南中 *Nanzhong*	24	129,308	丙盛		南中為宮，安度商，丙盛徵	7	
丙盛 *Bingcheng*	25	172,410		安度	丙盛為宮，屈齊商，安度徵	6	
安度 *Andu*	26	114,940	屈齊		安度為宮，歸期商，屈齊徵	6	
屈齊 *Quqi*	27	153,253		歸期	屈齊為宮，路時商，歸期徵	6	
歸期 *Guiqi*	28	102,169	路時		歸期為宮，未育商，路時徵	6	

A	B	C	D	E	F	G	J
路時 *Lushi*	29	136,225		未育	路時為宮，離宮商，未育徵	6	
未育 *Weiyu*	30	90,817	離宮		未育為宮，凌陰商，離宮徵	8	
離宮 *Ligong*	31	121,089	凌陰		離宮為宮，去南商，凌陰徵	7	
凌陰 *Lingyin*	32	161,452		去南	凌陰為宮，族嘉商，去南徵	8	
去南 *Qunan*	33	107,635	族嘉		去南為宮，鄰齊商，族嘉徵	8	
族嘉 *Zujia*	34	143,513		鄰齊	族嘉為宮，內負商，鄰齊徵	8	
鄰齊 *Linqi*	35	95,675	內負		鄰齊為宮，分動商，內負徵	7	
內負 *Neifu*	36	127,567	分動		內負為宮，歸嘉商，分動徵	8	
分動 *Fendong*	37	170,089		歸嘉	分動為宮，隨期商，歸嘉徵	6	
歸嘉 *Guijia*	38	113,393	隨期		歸嘉為宮，未卯商，隨期徵	6	

A	B	C	D	E	F	G	J
隨期 *Suiqi*	39	151,190		未卯	隨期為宮，形始商，未卯徵	6	
未卯 *Weimao*	40	100,794	形始		未卯為宮，遲時商，形始徵	6	
形始 *Xingshi*	41	134,392		遲時	形始為宮，制時商，遲時徵	5	
遲時 *Chishi*	42	89,595	制時		遲時為宮，少出商，制時徵	6	
制時 *Zhishi*	43	119,460	少出		制時為宮，分積商，少出徵	8	
少出 *Shaochu*	44	159,280		分積	少出為宮，爭南商，分積徵	6	
分積 *Fenji*	45	106,187	爭南		分積為宮，期保商，爭南徵	7	
爭南 *Zhengnan*	46	141,582		期保	爭南為宮，物應商，期保徵	8	
期保 *Qibao*	47	94,388	物應		期保為宮，質末商，物應徵	8	
物應 *Wuying*	48	125,850	質末		物應為宮，否與商，質末徵	7	

A	B	C	D	E	F	G	J
質末 *Zhimo*	49	167,800		否與	質末為宮，形晉商，否與徵	6	
否與 *Fouyu*	50	111,867	形晉		否與為宮，夷汗商，形晉徵	5	
形晉 *Xingjin*	51	149,156		夷汗	形晉為宮，依行商，夷汗徵	6	
夷汗 *Yihan*	52	99,437	依行		夷汗為宮，色育商，依行徵	7	
依行 *Yixing*	53	132,582	色育		依行為宮，謙待商，色育徵	7	
色育 *Seyu* (宮 1) [Do]	54[256]	176,776		謙待	色育為宮，未知商，謙待徵	6	

[256] *Seyu*, the 54th tuning-standard, is the pitch with the Large Number nearest to *Huangzhong*. The comma is thus reduced. The ratio between *Huangzhong* and *Seyu* is called the Jing Fang Comma (*Jing Fang Yincha* 京房音差). From *Seyu* a Seven-Note scale is generated, which ends the cycle at sixty tuning standards. Thus, there is a matching, nearly identical scale in the first and last seven pitches of the table.

A	B	C	D	E	F	G	J
謙待 *Qiandai* (徵 4) [Sol]	55	117,851	未知		謙待為宮，白呂商，未知徵	5	
未知 *Weizhi* (商 2) [Re]	56	157,134		白呂	未知為宮，南授商，白呂徵	6	
白呂 *Bailü* (羽 5) [La]	57	104,756	南授		白呂為宮，分烏商，南授徵	5	
南授 *Nanshou* (角 3) [Mi]	58	139,674		分烏	南授為宮，南事商，分烏徵	6	
分烏 *Fenwu* (變宮 6) [Si]	59	93,116	南事		分烏窮次，無徵，不為宮 Generation ceases.	7	

A	B	C	D	E	F	G	J
南事 *Nanshi* (變徵 7) [Fa#]	60	124,154	None	None	南事窮，無商、徵，不為宮 Generation ceases.	7	
					Total:	366	

The Practice of Observing *Qi*

The documentation of the practice of Observing *qi* began in the Western Han with Jing Fang. Since "Heaven models shadow, and Earth models echoing sounds. These are the pitch pipes [measures]. When the *yin* and the *yang* are harmonized, the shadow reaches [a certain length]. When the pitch pipes and the *qi* correspond, the ashes fly off." The little information provided in the treatise tells us that Obeserving qiwas a sonic and visual spectacle, which also included musical performance (*yue* 樂). We are told that the experts of the Eight Capacities oversaw the quality and correctness of the musical instruments, listened to the Music, and examined the conditions of the gnomon shadow, the tuning measures, and the ashes on the ground. Moreover, the musical performance is characterized as being clear during the winter solstice, and turbid during the Summer solstice. The meaning of this characterization of sound remains unclear, but it is one of the only indications we have of a specific quality of sound. It certainly shows that different calendric periods included different timbers, or use of gamut, to the point where in the solstices they were either entirely clear or entirely turbid.

The experts reported the results to the emperor. In the case of unfavorable results, when there was no correspondence between the calendar and pitches, Jing Fang recommends divination. The emperor's participation during the solstices manifested the Heaven–Man–Earth cosmological theory, prevalent during the Western Han. Jing Fang offered a method that allowed the ruler to claim attunement, musically and cosmologically, with cosmic processes via the transference of *qi* from the earth.

Conclusion

Jing Fang's relation to *Changes* interpretation and prognostications on the one hand, and to astrology and the calendar on the other, caused his accomplishments and influence in acoustics to drown in accusations of mysticism (*shenmi zhuyi* 神秘主義) and superstition (*mixin* 迷信) in 1950's Chinese scholarship.[257] On the other hand, Chen Yingshi 陳應時 insists on divorcing Jing Fang's tuning method from any type of non-positivist context, and asserts that his method of calculating the sixty tuning standards was not an extension of his *Changes* interpretation method, which had no direct influence on his calculations. He specifically argues against the use of the *Gua qi* hexagram interpretation method, which is a subcategory of a larger system called Images and Numbers (*Xiangshu* 象數), popular in the Western Han, and sometimes associated with Jing Fang's acoustical work.[258]

The debate over the correlation of Jing Fang's musicological calculations to specific hexagram interpretation systems is beyond the scope of the current chapter. Nevertheless, while the *Sanfen Sunyi* system is strictly used for acoustic purposes, and while any direct relation to systems of hexagram interpretation is difficult to ascertain, we should not disregard similarities to fundamental cosmological ideas that dominated much of the

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[257] Yang Yinliu 楊蔭瀏, *Zhongguo gudai yinyue shigao* 中國古代音樂史稿 (Beijing: Renmin Yinyue 人民音乐, 1981), 131–32.

[258] For a detailed summary of the debate over Jing Fang's reliance on the *Gua Qi* interpretation theory for his calculations see Huang Lixing 黄黎星, "*Zai Lun Jing Fang 'Liushi Lü' Yu Gua Qi Shuo* 再論京房'六十律'與卦气說," *Huangzhong* (*Wuhan Music Conservatory*)《黄钟(武汉音乐学院学报)》, no. 2 (2010):121-127, and Chen Yingshi 陳應時, "Jing Fang Liushi Lu_ Zai Ban 京房六十律_三辨," *Huangzhong* (*Wuhan Music Conservatory*)《黄钟(武汉音乐学院学报)》 2 (2010): 113–20. Also, for a third point of view on the relation between hexagram interpretation and Jing Fang's tuning method see Gu Jie 谷杰, "*Jing Fang yixue de Najia, Nazhi fa ji qi yingyang wuxing yu 'liushi lü' de shenglü fa* 京房易学的纳甲、纳支法及其阴阳五行与'六十律'的生律法," *Journal of Zhejiang Vocational Academy of Art* 浙江藝術職業學院學報 13, no. 1 (2015): 35–44.

underlying theories behind technical achievements related to the cosmos, regardless of *Changes* interpretation theories. Some of these include the alternating character and mutual generation of *yin* and *yang*, the belief that *qi* is detectable, and the general preoccupation with synchronizing cycles.[259] Moreover, Jing Fang's reliance on numbers (*shu* 數) to provide a detailed theory on the functioning of sound throughout the year is tangential to the use of numbers in the *Xiangshu* interpretation method. Both use *shu* as part of the philosophy that explicated the workings of the cosmos.

Moreover, the text contains numerous indications that tie his acoustic accomplishment to the world of cosmology: A) The sixty tuning standards were used in the Department of Astrological Observers (*houbu* 候部). B) The sixty tuning standards and the creation of the *zhun* were put to practice through the ritual of Observing *Qi*, which focused on the idea that *qi* traveled through the ground and manifested as sonic vibrations, traveling up pitch pipes undisturbed. C) The description of the ritual, the participations of the Emperor, and the duties of the officials of the Eight Capacities only further magnify the cosmological significance of sound and accurate tuning.

Kalinowski's argument regarding the tuning formulas of the twelve tuning-measures found in the excavated scrolls of Fangmatan (239 B.C.E.) applies here as well:

> The primacy accorded to the chromatic scale of the tuning standards by the statement of harmonic numbers and the system of generations by alternating fifths and fourths cannot be explained without appealing to the more numerological than strictly musical conceptions developed by the cosmologists and Chinese calendars

[259] See Sivin, "Cosmos and Computation."

from the unalterable principle of agreement between the natural progression of the twelve semitones within the octave and the deployment of seasonal blowing of [pitch pipes] in the course of a year.[260]

Much has been written on music as an aid to the cultivation of morality on the part of the ruler and the people, and on the sounds of a dying state. Not much, however, has been dedicated to the importance placed on the role of tuning standards and the cosmological significance sound had in the mind of Han elite society. Jing Fang's calculations were probably not of use in everyday music, but there are several indications of the use of this tuning on ritual instruments. The *HHS* "Treatise on Tuning Standards and Mathematical Astronomy," translated above, provides an example in which a person who knew cosmological tuning could oversee tuning musical instruments for ritual performance. Even if these tuning methods were not applied to all musical performances, they might still have been applied to some. People do the best they can to keep their voices and string instruments in conformity with these tuning ideals, but keeping them steady is quite difficult, especially when dealing with commas and micro intervals like Jing Fang's. Astrologers and astronomers seem to have been the main utilizers of these calculations, and once they insured that the instruments were satisfactorily tuned, the sounds could be deemed correct (*zheng* 正). While our modern perception of music gives meaning only to the final product – a musical piece – in early China the components of music were loaded with meaning, and it is their accurate cosmological correspondence (*xie* 協) that created music.

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[260] Kalinowski, "Musique et Harmonie Calendaire," 136–37.

Part Three

THE FANTASY OF CONTROL: SOUND AS A FOUNDATION FOR WEIGHTS AND MEASURES STANDARDIZATION

In Part One of this work, I mapped a process by which sound assumed cosmological traits. I also showed that in the case of hexagram divination, sound also shed its original trait – namely audibility. In the second chapter, I focused on Jing Fang's mission to calculate the sound emitted by each few days in the year's cycle and mark its measure on his instrument- the *zhun*. His goal was to allow better synchronization between the realms of heaven and earth. These are instances when sound is used as technology in what we would consider today exo-musical settings. The role of the *Huangzhong* pitch pipe as the basic measure for the emperor's act of standardizing weights and measures is another such exo-musical activity, especially since no sound was required to do it – only the length, volume, and weight of this conduit of *qi* mattered. However, such a dichotomous view did not exist in early China. Major acts of cosmic synchronization required the measuring of sound, and weights and measures certainly belonged to this category.

Cosmic synchronicity also provided the claim to the mandate and painted an image of control and power of the emperor. The *Han shu* "Treatise on Tuning Standards and Mathematical Astronomy"[261] establishes the *Huangzhong* pitch pipe as the basis for the

[261] Hereafter *Han shu* LLZ

standardization of metrological units in the realm. This was another tributary of the cosmos-sound-man triad so immanent in the conceptualization of sound in early China. This chapter explores the idea of imperial control in the Western Han (206 B.C.E. – 9 C.E.) Xin dynasty 新 (9-23 C.E.), by examining the connection between sound, metrological practices, and metrosophical theory – the ideologies and philosophies that provided cosmological meaning to metrological choices. One aspect of control is tangible, expressed on the ground by the allotment of local rulership to relatives, campaigns of territorial expansion and the like, while another aspect of control is the ideology mirrored to and accepted by a public. This included the notion of power over Heaven led by a deciphering, quantifying, and displaying of cosmic patterns and their synchronization with earthly living. Technology is one very useful way to express the multifaceted concept of power. Above, it meant controlling calculation of the almanac, predicting eclipses, predicting weather, and tracking celestial motions, among others. Below, cosmic legitimation included standardizing weights and measures, performing ritual music and dance in the court, and symbolic architectural designs, among various other demonstratives. In early Chinese thought, the nexus between these two realms manifested in *Huangzhong*. Its qualities as a sound, vibrating *qi*, number, measure, standard, and object governed the connection between the cosmos and the microcosm of human society; its general function metaphorically resembled that of the emperor.

In its wish to display its authority, the Western Han and Xin administrations created an ideology of control to legitimate its mandate and emanate its power. We witness the fantastic aspects, for example, in things like the cosmological superstructure of cycles behind

mathematical astronomy, which caused Han astronomers to make avoidable errors.[262] In the case of weights and measures the problem is similar. But while astronomical synchronization could be accomplished by officials within the confines of the court, the standardization of weights and measures, based on the dimensions of the *Huangzhong* pitch pipe, required the populace to recognize and adopt the issued standards. What happens when theory and practice do not match; when the rhetoric in the textual sources makes it appear as though they did; and when the image of control that arises from all of the above is that of a fantasy of absolute control?

Sound, Weights, and Measures

Hans Ulrich Vogel defines "metrosophy" as the inquiry into weights and measures in relation to cosmology in premodern civilizations:

> With the word "metrology", the 'science of weights and measures' is meant, but when referring to premodern periods rather the term 'knowledge of weights and measures' is appropriate. In a more concrete sense, metrology is the art of calculation with number, weight and measure units in the economic, fiscal and scientific domains. "Metrosophy" may be defined as 'number speculation within cosmological philosophemes', but from a more inclusive perspective this is certainly too limited a definition, because also the relationships between magical, religious and political thought on the one hand and metrology on the other have to be taken into account.[263]

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[262] As Sivin notes, Han astronomers, who were far from "playing primitive mathematical games for their own amusement, but [were] scientists at work on problems of practical consequence...were led to an eclipse prediction method of low accuracy by their metaphysical commitment to simple linear techniques." Sivin, "Cosmos and Computation," 52.

[263] Hans Ulrich Vogel, "Aspects of Metrosophy and Metrology during the Han Period," *Extrême-Orient Extrême-Occident*, no. 16 (1994): 135.

The interrelationships between heaven and earth, man and cosmos, were the cornerstone of the Western Han and Xin court ideology. They made the measuring of sound one of the ultimate showpieces of control, pervading ritual — the power to cultivate the self, society, and communicate with gods and ancestors; astronomy and calendric calculations — the power to predict the passing of time, motions in space, and seasonal changes; weights and measures — the power to oversee commerce, manufacturing, and coinage across the territory by providing state regulations. The ideas behind these actions were based on a theory of resonance. Such was the ritualistic display of the micro/macro cosmos design, which had the emperor seated at its nexus, and the true measurement of the fundamental cosmic sound as his responsibility.

Several pre-Qin texts from the *Discourses of the States* (*Guoyu* 國語) to official histories such as the *Han shu* group together standards of tuning, length, volume, and weight (律、度、量、衡). The *Han shu* LLZ is the first to tie weights and measures with sound in a systematic fashion. It presents metrological data combined with a coherent metrosophical theory originating in cosmic sound.[264] The dissemination of metrological standards required the administration to take three major actions: adopting units of length, volume and weight; constructing durable vessels that embodied these standards and served as examples for copying; circulating models, together with a system of official oversight meant to ensure their availability to the public and correct use in commerce, taxation, and other activities.[265]

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[264] Vogel 1994, 136–36

[265] Michael Loewe, *Problems of Han Administration: Ancestral Rites, Weights and Measures, and the Means of Protest*, China Studies, volume 33 (Leiden ; Boston: Brill, 2016), 156.

Several early texts discuss the general importance of the adherence of musical instruments to measures, but only few discuss specific instruments (other than pitch pipes) as exemplars for embodying correct weights and measures. One of the earliest texts to incorporate this idea is the "Discourses of Zhou" chapter in the *Discourses of the States*. In a cryptic exchange between Ruler Jing of Zhou 景王 (r. 544–520 B.C.E.) and Lord Mu of Shan 單穆公, the ruler was about to cast a *Wuyi* 無射 bell but wished to change it to *dalin* 大林 and asked Lord Mu about it. Lord Mu then reprimanded Jing at length on the gravity of using the correct pitch name to designate the correct bell size:[266]

> 是故先王之制鍾也，大不出鈞，重不過石。律度量衡於是乎生，小大器用於是乎出，故聖人慎之。今王作鍾也，聽之弗及，比之不度，鍾聲不可以知和，制度不可以出節，無益於樂，而鮮民財，將焉用之！[267]

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[266] Whether Wang wished to change the bell's name alone, its pitch, or its placement within the scale is unclear, therefore I leave it as a general statement. The Chinese sentence is extremely concise and does not reveal much information. "The ruler was about to cast *Wuyi*, but/and make it *Dalin*" 王將鑄無射，而為之大林. Nevertheless, it has been a source of great speculation among scholars, possibly because it is one of the only examples of a specific action having to do with pitches in early Chinese sources. Scholars have offered several interpretations to this sentence. Niu Longfei 牛龍菲 suggested that it refers to the second of the two striking points (*gu* 鼓 and *sui* 隧) on a two-toned bell. According to him, the main striking point (*gu* 鼓) would have been the *Wuyi* pitch, and the second – *Dalin*. However, since there is no other reference to such a pitch name, he suggests this was another name for *Linzhong* 林鍾. Wang Hongjun 王洪軍 and others rightfully refute this, mainly since the twelve tuning standards are listed in the following section of the text, and *Linzhong* appears there by its name and not as *Dalin*. Also, Qinxi 秦序 argues that a *Wuyi*/ Linzhong interval on a bell would be acoustically impossible to create. Wang Hongjun 王洪軍 2007, 326–32. James Hart argued that *Wuyi* refers to a bell-series, rather than a single one. Thus, the ruler wished to create a *Wuyi* series "and make for it a great *lin* bell," which, he explains, would have been a grand low-pitched bell that would fit within the series, possibly as its lowest pitch. Although this is a noble effort, it overstates the musical information provided in the text. James Hart, "The Discussion of the Wu-Yi Bells in the Kuo-Yü," *Monumenta Serica* 29 (1970): 404. However, Duke Mu's reply does imply that a great amount of resources would have been used to create it, thus depriving the people of their wealth.

[267] ann. Xu Yuangao 徐元誥, *Guoyu Jijie* 國語集解 (Beijing: Zhonghua Shuju, 2002), 107-109.

> Thus, when the sage rulers of old established[268] bell [measures], in size it would not exceed a *jun*;[269] in weight it would not surpass a *shi*. Because it is from this that tuning[270] [using] length, volume and weights was born,[271] and it is from this that the use of smaller and larger vessels arose. Sages were scrupulous about measures. Today, when you (the ruler) make a bell, when listening [to its pitch], it is not discernible; when examining [its size], it is not measurable. The bell sound is impossible to harmonize. The measures do not derive from the standard. It is useless for joy/musical performance and deprives the people of their wealth.[272] What use is it?![273]

In fact, Lord Mu berates Ruler Jing for his moral offenses, bad aesthetic predilections, and monetary extravagance in ritual music performance (e.g. "One listens to the music and quake, watches its beauty and becomes dazzled" 聽樂而震，觀美而眩). He relies on the argument that the internal method for correct rulership over society begins with sharp hearing (*cong* 聰) and keen sight (*ming* 明). If the ears cannot sustain the bell's sound (*sheng* 聲) and the eyes cannot grasp its measures (*du* 度) because both are false, then great confusion arises. These qualities are extended outward to the actions of the ruler and the

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[268] I read *zhi* 制 as "establishing measures," based on its meaning in other texts relating to weights and measures, and the commentary to this section in the *Guoyu Jijie*.

[269] *Jun* 鈞 often refers to a weight unit, but its meaning is unclear. According to Xu Yuangao′s (徐元誥 1876-1955) commentary, it is the name of a measuring method called *jun yin* 鈞音, which included a long wooden plank measuring 7 *chi* 尺, with a string tied to it.

[270] I interpret *lü* 律 here as the action one takes to standardize other measures in accordance with the principals of sound measures. Scholars often translate *lü du liang heng* 律度量衡 as four distinct categories, but this approach ignores the supremacy of sound in the contexts in which this tetrad most often appears.

[271] The commentaries still try to coerce the meaning of the text, arguing that the *Huangzhong* pitch pipe, the *yue* volume, and the millet grain method are at the basis of this statement. Nothing in the text indicates this

[272] Much of this section of the "Discourses of Zhou" discusses the consequences of unnecessary expenditures by rulers in making heavy coins, or bells. The overall argument is that this extravagance comes at the expense of the people′s wealth, and therefore it is a sure recipe for losing the throne.

[273] This last sentence could reference the *Analects* 16.1 危而不持，顛而不扶，則將焉用彼相矣？

response of the minds of the people (*min xin* 民心). The wise ruler corrects weights and measures, and the people follow him willingly:

> 夫耳內和聲，而口出美言，以為憲令，而布諸民，正之以度量，民以心力，從之不倦。
>
> This 'harmonizing of sound within the ears and producing elegant speeches through the mouth' generates decrees that spread among the people. Correct these by using weights and measures[274] and the people, in mind and might, will follow them tirelessly.

When the ruler's hearing and sight are not harmonized, his essence (*jing* 精) is gone, and the *qi* that gave him his speech and clarity of sight dissipates and there is no harmony. His words become arrogant; his sight confused; measure become corrupt (*you guo te du* 有過慝之度), correct names (*ming* 名) are distorted; the people will not know strength and their minds will drift away from the ruler. They will cease to use his tools, and the state will be in great danger. The actions of Ruler Jing led him in this direction.

According to this passage, the internal world of the ruler and the external world of sound and measures are inseparable and not mutually exclusive – they form a type of symbiotic relationship. The anecdote presents bells as embodying correct measures, but not necessarily as standardizing tools in their own right.[275] We have no material evidence that shows bells served as measuring standards, although they are often referred to in discussions

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[274] D*u liang* 度量 literally mean "degrees and capacities."

[275] Falkenhausen suggests that this passage may refer to the bell as the standardizer of weights and measures, but "On the other hand, the fundamental unit of measurement as envisaged by the *Guo Yu* passage might equally likely have been a pitch-pipe or a vibrating string that was tuned to the pitch of a bell." Falkenhausen, *Suspended Music*, 315.

on tuning standards. Several texts use the term bell standards (*zhong lü* 鍾律) to refer to experts in musical temperament long after bronze bells fell out of favor and pitch pipes.[276] Falkenhausen surmises that linking sound and measures was a new idea in the period the text was written and may have been influenced by the rise of correlative cosmology in the late Warring States period. While this very initial link to correlative thinking through weights and measures may be correct for this account, the dating of the *Discourses of the States* has been inconclusive.[277] The relation between sound, weights and measures, and cosmology is much more evident in the account following this one, in which Ruler Jing, once more, was about to cast *wuyi* and asked about tuning standards:[278]

> 王將鑄無射，問律於伶州鳩。對曰：「律所以立均出度也。古之神瞽考中聲而量之以制，度律均鍾，百官軌儀，紀之以三，平之以六，成於十二，天之道也。夫六，中之色也，故名之曰黃鍾，所以宣養六氣、九德也。
>
> The ruler was about to cast *wuyi* and asked Lingzhou Jiu about the tuning standards. He replied: "the tuning standards establish the *yun* [device] and produce measures. The divine blind [musicians] of old examined and identified sounds,[279] and measured them with tools. They measured[280] tuning standards, standardized bells, and the many offices regulated their

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[276] The terms itself alludes to the idea that in antiquity, bells were used as standards for pitch as well as weights measures. By the Han, this was no longer the case, but *zhong lü* still appears in texts that refer to experts in temperament. The *HHS* LLZ is one example of such usage. See its translation in Part Two.

[277] Falkenhausen dates the *Discourses of the State* to the late Warring States. Boltz dates the text to the early Warring States, some time after c. 425 B.C.E., with the *Discourses of Zhou* as one of the earlier sections. Knechtges and Chang argue that parts of it were written in the Warring States period. William G. Boltz, "Kuo Yü 國語." David R. Knechtges and Taiping Chang, eds., *Ancient and Early Medieval Chinese Literature: A Reference Guide*, vol. 1, Handbook of Oriental Studies. Section Four, China, Handbuch Der Orientalistik, v. 25-1--25-4 (Leiden [Netherlands] ; Boston: Brill, 2010), 309.

[278] I am uncertain whether these are two occasions, or whether this is a retelling of the same occasion with a different narrative. Linzhou Jiu conceptualizes a host of cosmological, numerological, and astronomical explanations, which do not appear in the first account.

[279] I translate 中 as "hitting a target," meaning here to be able to identify a pitch. See the term *zhong lü* 中律 – identifying tuning standards, in the translation of the *HHS* LLZ in Part Two this dissertation.

[280] *Du* 度 here is specific, and because it refers to a linear measures in the case of tuning standers, it probably points to the length measure of pitch pipes or strings.

> etiquette. They systematized it with three, equalized it with six, and perfected it through twelve. This is the Way of heaven. As for the six, they are the color of the center. Thus, it is named *Huangzhong*, and with it [they] comprehensively develop the six *qi*, and the nine virtues.[281]

The numerological association to the number three leaves little doubt that Lingzhou Jiu, unlike Duke Mu of Shan, is talking about the *sanfen sunyi* method of calculating interval ratios.[282] This method applies to the ratios linear lengths such as in strings and pipes but does not apply to bells. Thus, his explanation places pipe length or string lengths as the origin of standardization and bells as their embodiment in sound. The striking differences between these two accounts suggests that they may have been written by different authors in different periods, but both still recognize the importance of bells as fixed embodiments of standardized measures. The *Han shu* LLZ, which I discuss below, forgoes these instruments, and focuses instead solely on numbers and pitch pipes.

The "Treatise on Tuning Standards and Mathematical Astronomy" in the *History of the Former Han*

As mentioned above, the *Han shu* LLZ is the first text to forge a systematic connection between metrology, sound, and cosmology. The treatise was edited by Ban Gu from a text

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[281] James Hart analyzes and provides a full translation of these two accounts, including the section on sound and astronomy. His is one of the first works in the English language, excluding Joseph Needham and Kenneth Robinson to investigate the role of sound and music theory in the early Chinese cosmological thought. Hart, "The Discussion of the Wu-Yi Bells in the Kuo-Yü."

[282] Three was the quintessential *yang* number from which the calculation of the tuning standard began. The number six refers to the number of the tuning standards in each set of the *yang lü* 律 and *yin lü* 呂 (or *jian* 間 as they are called later in this account. For a discussion on the development of the twelve *lü* see Part One); and the number twelve likely refers to the combination of the two sets, later known as the twelve tuning standards (*shier lü* 十二律).

written by a group of over a hundred specialists in tuning standards led by Liu Xin. In the opening of the treatise, Ban Gu informs readers that his editing was selective, and omitted some information that he deemed incorrect.[283] The name of the original text may have been *The Book of Temperament Zhong Lü Shu* 鍾律書.[284]

Liu Xin chose to include a section on weights and measures in a treatise focusing temperament calculations and mathematical astronomy. Given their importance in converting theory into practice, at least in the ritual sense, one may wonder why weights and measures were left out of the title. In my view, this indicates that, in Liu's eyes, weights and measures did not stand on their own. They were a sub-section of a conversation about numbers and the calculation of tuning standards (*lü* 律). They underlay the implementation and expansion of the standards into practical use. This is also evident in Liu Xin's outline of this portion of the treatise, which he organized according to five categories: numbers, sound, length, volume, and weights. He dedicated the second portion to mathematical astronomy.[285] The *Shuowen jiezi* glosses *lü* 律 (*rut)[286] as "equitably distributed" (*jun bu* 均布), possibly implying a standard.[287] In the musical sense, *lü* referred to the agreed-upon measures of the *Huangzhong* pitch pipe, and from there – standardizer for weights and measures. Weights and

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[283] Michael Loewe, "Ban Gu: Copyist, Creator and Critic," *Bulletin of the School of Oriental and African Studies* 78, no. 02 (June 2015): 342–43.

[284] Loewe, *Problems of Han Administration*, 247–50. Loewe's translation of *zhong lü* as "bells and pipes" is literal, but the meaning of the compound simply means temperament, or tuning standards.

[285] The *HHS LLZ* does not discuss weights and measures. They are itemized in the exposition with tuning standards and mathematical astronomy as categories that are unified by the use of numbers. See Part Two for the annotated translation of the *HHS* LLZ.

[286] Old Chinese reconstructions follow the system in Baxter and Sagart, *Old Chinese*, 374.

[287] Xu Shen 許愼, *Shuowen jiezi xin ding* 說文解字新訂 (Beijing: Zhonghua Shuju 中華書局, 2002), 116.

measures were the implementation of numbers and standards, and their expansion into practical use, producing in a display of power, and proof of cosmic legitimacy.

The opening statement of the treatise credits the act of standardizing to the venerated past thearchs Yao 堯 and Shun 舜, and associate Wang Mang, by his contribution to standardization and his commissioning of this treatise, to their glory. It opens with a quotation from the *Book of Yu* 虞書 on Shun's act of standardization after Yao conferred the rule of the world upon him: "The *Book of Yu* says: 'Then, he made the standards of tuning, length, volume, and weight, correspond' and thus made near and far, uniform, and used them to establish the trust of the people"《虞書》曰「乃同律度量衡」，所以齊遠近立民信也。[288] Liu Xin then rushes through the far past to the Early Han, also adding a quotation from the *Analects* in which Confucius advises: "Decide on standard weights and measures after careful consideration, and re-establish official posts fallen into disuse, [elevate worthy people], and government measures will be enforced everywhere."[289] In the full passage in the *Alanects*, Confucius surveys the actions of great thearchs, the downfall of the Shang and the rectification of the Zhou, ending with the aforementioned advice. The quote echoes what Liu Xin does in his treatise. It seems this quote was meant not only to link Wang Mang directly to the venerated ancient thearchs, but also to invoke Confucius' authority.

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[288] When analyzing the inscription of the *Jia Liang* Hu, discussed below, Loewe suggests that the lineage association between Shun (i.e. Yu) and Wang Mang possibly aimed to distance Wang Mang from the Liu family of the Western Han, who was associated with the sage emperor Yao. Loewe, *Problems of Han Administration*, 227 n.52. The quotation from the *Book of Yu* that appears here suggests a bestowal of the heavenly mandate upon someone external to Yao′s family, based on his strength of character. Perhaps this is what Wang Mang wished to imply, or adjust, in the story of his own new rule.

[289] Analects 20.1; D. C Lau, trans., *The Analects (Lun Yü)* (Hong Kong: Chinese University Press, 1983), 200. The squared brackets are my own addition, since those characters only appears in the *Han shu* citation.

Michael Loewe rightfully notes that Liu Xin deliberately ignores the substantial accomplishments of the first emperor of the Qin dynasty (秦 221-206 B.C.E.) in standardizing weights and measures. Instead, Liu continues his survey with the early Western Han efforts by Zhang Cang 張蒼, Marquess of Beiping 北平候 (d. 152) and advisor to Gaozu (r. 202-195) on tuning standards and mathematical astronomy. He then recounts Emperor Wu's (r. 141-87) examination and reform (*kaozheng* 考正) of the Office of Music (*Yue guan* 樂官). These references tie together musical performances, cosmological tuning standards, astronomy and weights and measures as belonging to the same realm.[290] After emperor Wu, Liu Xin skips the next eight Han emperors, landing on the reign of Wang Mang of the short-lived Xin dynasty (r. 9-23 C.E.). He praises Wang for commissioning Liu to gather over a hundred other erudites and write the treatise we are about to read.

Liu Xin's five categories follow the chronological overview. He begins with the significance of numbers. This section is important because it concisely sums up the connections between all five categories, and how they are to be used. Numbers (*shu* 數)[291] sit at the foundation of everything in the world, and originate in the number for *Huangzhong*.[292] The entire section solidifies the claim that the calculations, methods, instruments, and the theories that Liu Xin presents throughout this treatise produce precise results. He argues that when we calculate things, we conform to the patterns of innate nature and destiny (所以算數事物，順性命之理也). Then, he provides a simple arithmetical

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290 The text doesn't directly speak of this connection, but I infer it from its place in the text, and the function of the tuning standards in the Western Han. I cannot see another reason for this concise mention of the Music Bureau here.

291 Sometimes the character *shu* 數, which means numbers, also refers to qualitative regularities.

292 This is similar to the *HHS* LLZ opening, which argues for the same proposition.

form of the *Huangzhong* tuning standard: multiplying the number 3 by 1 once and by itself eleven times results in 177,147. This number resonates with the regularities of the twelve branches (十二辰) used in to record the motions of the moon, sun, the cycle of the seasons, and so on.[293] In this way, we are told, the number of *Huangzhong* encompassed the five regularities (*wu shu* 五數).[294] This is the synthesis between numerology, cosmology, sound, and astronomy. Liu then also gives the dimensions for short, thin, bamboo pipes tube used in a hexagram divination technique associated with the tuning standards and based on the *Book of Changes*.[295] The introduction to measuring instruments is also placed in this first section on numbers:

> 夫推曆生律制器，規圜矩方，權重衡平，準繩嘉量，探賾索隱，鉤深致遠，莫不用焉。
>
> This [method of] inferring mathematical astronomy by generating tuning standards produced [measuring] instruments. The compass and the carpenter's square, weights and balances, the water level and marking line and the excellent volume.[296] [we] explore the mysterious and search the hidden, investigate deeply and reach far. No one fails to use it.

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[293] In practice, this number is divisible by 3 twelve times without a remainder, thus satisfying the *Sanfen Sunyi* method for generating tuning standard ratios. For four different meanings of the character *li* 歷 see Sivin, *Granting the Seasons*, 39–40.

[294] Yan Shigu 顏師古 cites Meng Kang 孟康 on the five regulations: "through this, the regularities of the transformation of the five phases and *yin yang* are complete." 五行陰陽變化之數備於此矣. I am uncertain how these come up to five. By the Western Han *yin yang*, the five phases, and *qi* were thought to work together, whereas before we see them discussed separately. This treatise incorporates all of these forces. See G. E. R. Lloyd and Nathan Sivin, *The Way and the Word: Science and Medicine in Early China and Greece* (New Haven: Yale University Press, 2002), Appendix.

[295] This is not a reference to pitch pipes. The diameter of the pipe is associated with the six *yang* tuning standards, beginning with *Huangzhong* and the *Qian* hexagram, while its length is associated with the *yin* tuning standards, beginning with *Linzhong* and the *Kun* hexagram. *Linzhong* is the standard that *Huangzhong* generates. The rest of this section discusses the dimensions of these short pipes in relation to hexagrams.

[296] The *zhun* 準 insures an even surface by using water, and the *sheng* 繩 are marking lines that direct the level of capacity. These were used together to measure volume contents. The volume measure was determined using a geometry formula, which I will discuss below.

After Liu Xin's assurances of meaningful action and ubiquitous use, he vouches for the precision of these measuring instruments and methods:

> 度長短者不失豪氂，量多少者不失圭撮，權輕重者不失黍絫
>
> when measuring length, not even a *hao* or *li* will be lost; in volume, not even a *gui* or *cuo* will be lost; and in weight, not even a single millet grain will be lost.[297]

This is a guarantee Liu Xin provides for his readers, wrapping together metrology, metrosophy, power, and legitimacy, as originating in the concept of sound. In this section, which is dedicated to numbers it is reflected in the number for *Huangzhong*. In the section on sound it is reflected in the five-note scale and its origin in the calculation and measuring of tuning standards. In the section on weights and measures, it is reflected in the measurements of the dimensions of the pitch pipe.

The five-note scale and eight timbres are the audible expression of sound in musical performance. On these Liu Xin gives the familiar declaration: "We use them (the five-notes) to make musical performance, and coordinate the eight timbers; subdue man's evil intentions, complete the rectification of his innate nature and change manners and customs." (*suoyi zuo yue zhe, xie bayin, dang jiang ren zhi xie yi, quan qi zheng xing, yi feng yi su ye* 所以作樂者，諧八音，蕩降人之邪意，全其正性，移風易俗也.)[298] He then provides glosses to each of their names. The next section relates the five-notes the twelve tuning standards, *yin*

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[297] *Hao* and *li* 豪氂 (sometimes written as *hao* 毫 and *li* 釐) describe extremely small length measures. *Gui* and *cuo* are extremely small volume measures.

[298] 移風易俗 is often associated with the cultivation power of ritual music performance in various Warring States and Han texts as early as the Cannon of Filial Piety (*Xiao Jing* 孝經), in the fourth century B.C.E. and often attributed to Confucius. Sometimes it can also relate to themes other than music.

yang and *qi*, and the myth of Ling Lun, followed by glosses for each of the twelve tuning standards names as well.[299]

After a cumbersome explanation of the principles of "heaven, earth and man" and relating *Huangzhong* to the astronomical system and the seasons, *yin yang*, *qi*, the twelve earthly branches, the hexagrams and the *Book of Changes*, the treatise finally reaches weights and measures.[300]

The Huangzhong pitch as a standardizer of length, volume, and weight

In the section on weights and measures a reference to *Huangzhong* opens each thematic section. It is recognized as the origin of each measuring standard:

1.	度者。。。本起黃鐘之長	Length[301]… originates in the length of *Huangzhong*
2.	量者。。。本起於黃鐘之龠[302]	Volume… originates in the *yue* volume of *Huangzhong*

[299] The technical and philosophical aspects of sound and the calendar are treated separately and in depth in the translation of the *HHS* LLZ in Part Two of this work. In the future, it will be interesting to compare between these two treatises (*Han shu* LLZ and *HHS* LLZ).

[300] The short section that explains the *Gai tian* astronomical system, reminds the reader that these are the principles that come together in music (*he yue yong ye* 合樂用也). Moreover, the image of performance it paints is very reminiscent of the *Zhouli* "Da Siyue" section, which discusses the education of young children in the rites and sacrifices, by teaching music. It might be a short-hand reference to the matching between *yin* and *yang* scales. I discuss that section in chapter 1 of this dissertation. Liu Xin writes: "When the *yang lü* and *yin lü* sing harmoniously with each other, by education the transformation is complete. Songs and Melodies use [tuning standards] in them." (*lülü chang he, yi yu sheng chang hua, ge zou yong yan* 律呂唱和，以育生成化，歌奏用焉。

[301] *Du* 度 is often translated as "degrees." Since the term originated in astronomy, and delineated the space between two positions, it is better, albeit less elegant, to translate it here as "length." In his discussion on the term *du* in Chinese mathematics, Cullen emphasizes that "In all the complex geometrical problems they discuss, the focus is always on the values and ratios of lengths." Cullen, *Astronomy and Mathematics in Ancient China*, 92.

[302] I think the reason for the use of the *yue* 龠 here, which is already a set measure, instead of *rong* 容 for volume, is due to the need to explain the principles measuring volume. This principle is presented in the second part of the sentence: "Applying length and numbers to determine its volume." (*yong du shu shen qi rong* 用度數審其容.) The geometrical formula for determining the area of a circle is presented later in thi chapter (p.148).

3. 衡權者。。。本起於黃鐘之重　　Weights and balances… originate in the weight of *Huangzhong*

The sections pertaining to each measuring unit type are constructed as follows: unit name; unit sizes; relation to *Huangzhong*; conversion of the standard measure to number of millet grains; relation between unit sizes (e.g. 10 *fen* equals 1 *cun*); measuring instrument for each dimension; glosses for unit and instrument names; title of office and official in charge.

Each section provides the number of millet grains used to measure the *Huangzhong* pipe in length, volume, and weight:

For length:

> 本起黃鐘之長。以子穀秬黍中者，一黍之廣，度之九十分，黃鐘之長。
>
> Originating from the length of *Huangzhong*. Using medium grains of black millet in pristine condition, with the breadth one grain measure 90 *fen*. This is the length of *Huangzhong*.[303]

For volume:

> 本起於黃鍾之龠，用度數審其容，以子谷秬黍中者，千有二百實其龠，以井水準其槩。”
>
> Originating from the *yue* of *Huangzhong*, use medium units and numbers to measure its volume. Using large grains of black millet in pristine condition, 1200 fill the *yue*.[304] Level the top with well water.

For weight:

> 本起於黃鐘之重。一龠容千二百黍，重十二銖，兩之為兩。

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[303] Compare translation with Loewe, *Problems of Han Administration*, 169.

[304] I elaborate below on problems scholars faces when interpreting this sentence, especially on the translation of *zhong zhe* 中者 as an indicator to the size of the grain (p. 165).

> Originating from the *Yue* of *Huangzhong*. The volume of a *yue* is 1200 grains; its weight is 12 *zhu*. Double it to make a *liang* (unit).

Each section also describes the appropriate measuring instruments for each unit type: bronze and bamboo foot-rules to measure length; a bronze vessel with compartments for each unit to measure volume; and a steelyard balance to measure weights.[305] All three types of measuring devices have been excavated from tombs or passed down and preserved over the years. The most striking example of a vessel that has been passed down is the *Jia Liang Hu* model vessel, which I will discuss below. Because there are those who attribute the inscription on the *Jia Liang Hu* to Liu Xin and the comparison between its dimensions and the information provided by Liu Xin here is of great importance to the discussion below, I provide a translation of two of the technical passages:[306]

> 其法用銅，高一寸，廣二寸，長一丈，而分寸尺丈存焉。用竹為引，高一分，廣六分，長十丈，其方法矩，高廣之數，陰陽之象也。

> The method: use bronze: 1 *cun* high, 2 *cun* wide, and 1 *zhang* long. The *fen, cun,* and *zhang* are in it. Use bamboo to make a *yin* foot-rule: 1 *cun* high, 6 *fen* wide, and ten *zhang* long. Its squared shape models the carpenter's square. The measures of its height and width are the counterparts of the *yin* and the *yang*.[307]

> 其法用銅，方尺而圜其外，旁有庬焉。其上為斛，其下為斗。左耳為升，右耳為合龠。

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[305] Excavated material shows that foot-rules across the Han dynasty were made with a wide variety of materials, and that the use of bronze to make rulers gained popularity only in the Eastern Han. The lasting and unchanging quality of bronze made it a perfect candidate for instruments made in the court.

[306] I do not provide a full translation for the section on weights and balances since I do not discuss their excavated equivalents later on.

[307] Another option for translation see Loewe, *Problems of Han Administration*, 170.

> The method: use bronze. A square with sides measuring 1 *chi*, and a circle around it. There are spaces between them.[308] Its top part is a *hu*; its bottom is a *dou*; the left ear is a *sheng*; the right ear is the *he* and a *yue*.

The form of the vessel is accompanied by a detailed cosmological commentary:

> 上三下二，參天兩地，圜而函方，左一右二，陰陽之象也。其圜象規，其重二鈞，備氣物之數，合萬有一千五百二十。聲中黃鐘，始於黃鐘而反覆焉，君制器之象也。
>
> The top is three and the bottom is two; heaven is three and earth is two; the circle and the incased square, the left is one and the right is two. These are the counterparts of *yin* and *yang*. Its circle resembles the compass and its weight is two *jun*. It provides the numbers for the *qi* of all things; combined it is 1520. Its sound is *Huangzhong*. It begins with *Huangzhong* and returns to it. This is the image of a ruler making a vessel.

These are familiar explanations from the Western Han for the numerological use of numbers. They include an emphasis on the numbers 2 and 3, representing heaven and earth, *yin* and *yang*. They use set expressions from the commentaries to the *Book of Changes*, such as "3 is heaven and 2 is earth" (*can tian liang di* 參天兩地), taken from the *Shuo Gua* 說卦 commentary, to express the association of odd and even numbers. The other half of the same sentence explains that two and three "establish [all] numbers/regularities" (*er yi shu* 而倚數). They also often discuss counterparts *xiang* 象 and hexagram lines (*yao* 爻).[309] As an example, when justifying his method for calculating tuning standards in the *HHS* LLZ, Jing

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[308] I elaborate on this sentence below, relying on Guan Zengjian 關增建, "Exploration of Liu Xin's Metrological Theory. Liu Xin Jisuan Lilun Guankui 劉歆計量理論管窺," *Journal of Zhengzhou Univeristy* 鄭州大學學報 36, no. 2 (2003): 125–30.

[309] I translate *xiang* 象 as counterpart, following Edward Schafer's translation, due to its emphasis on a relationship based on extension, rather than reflection between celestial and earthly phenomena. There are sections in this treatise where Liu Xin refers specifically to the *xiang* 象 in hexagram interpretation, in which case it can be translated as "image."

Fang says: "*yang* takes round as its shape. Its nature is dynamic. *Yin* takes square as its shape. Its nature is quiescent. Those that are dynamic number three; those that are quiescent number two… All of these are a matter of odds and evens (參天兩地), round Heaven and square earth."[310]

According to the text, the pitch of this five-compartment vessel is that of *Huangzhong*. This is quite a tall order, and seems like an unachievable requirement, especially since the experts could not tinker with any of the standardized sizes, by definition of the vessel's purpose. The reference seems to mean that all the measures initially derive from the dimensions of *Huangzhong*.[311]

The depiction of the steelyard balance is more intricate. These sections also involve more cosmological concepts than the sections on length and volume, especially in the glosses of the different weight units, which focus on the seasonal changes, and transformations of *yin yang* and divisions of *qi* throughout the year, lunar months, and the twelve branches. The formula of the prior two units of measures differs from this one. For example, weights are explained by a system of five derivations:

> 權與物鈞而生衡，衡運生規，規圜生矩，矩方生繩，繩直生準，準正則平衡而鈞權矣。是為五則。
>
> The evenness between weights and objects created the steelyard balance; the bending (?) of the steelyard balance created the compass; The roundness of the compass created the carpenter's square; the squareness of the carpenter's square created the marking lines; the marking lines

[310] See Part Two for the annotated translation.

[311] I do not know of any attempts to sample and measure the sound of the of the vessel (main body and smaller compartments).

> created the leveling [of water]; when the leveling is correct, then the balances are steady, and the weights are even. These are the five tools.[312]

Over the years scholars have discovered material evidence, which corroborate that the *Han shu* LLZ measures are not merely a theoretical discussion. I will examine it next.

Archaeology

In the cases of sound and music, and of weights and measures, archeological artifacts are some of our most reliable evidence. Thanks to the growing collection of relics and excavated material we are able to reexamine the link between theory and practice.[313] In this section I examine three surviving measuring tools: a large measuring vessel dated to the Xin dynasty, containing all five standard measures, called "Auspicious Measuring Vessel" (*Jia liang hu* 嘉量斛); a bronze ladle, also dated to the Xin dynasty, identified as a *yue*, and several foot-rules dated to the Western Han and the Xin.

Loewe has argued that "no more than a few of the examples of weights and measures that we possess were deliberately made in sizes that corresponded precisely with the units of the scales of measures that was in use; and certainly in the days of the Qin, it would seem that attempts to do so did not always produce results that were accurate."[314] Others have argued that foot-rules tend to show great uniformity, not only in the Han, but from the end

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[312] I also rely on Vogel's translation in Vogel, "Aspects of Metrosophy and Metrology during the Han Period," 139–40. Vogel translates *ze* 則 as "patterns," but I could not find a reason for this. As in other discussion regarding this term in the context of weights and measures, it refers to the instruments in use.

[313] Some artifacts have not been excavated, but passed down, like in the case of the *Jia Liang Hu*.

[314] Loewe 2016, 156

of the Warring States until the Eastern Han.[315] Though a general standard measure of around 23.1cm may have existed since the late Warring States, the collected data from recently excavated foot-rules and volume measures indicate no strict uniformity beyond the court exemplars. What level of precision, then, did the administration expect? Were vessels that varied from the standard still considered lawful?

The *Yue* 龠 volume in the *Jia Liang Hu* 嘉量斛

One of the most celebrated vessels made to display standardized metrological units is the *Jia Liang Hu*. Dated to 9 C.E., the first year of the Xin Dynasty (新潮 9-23 C.E.), it glorifies its monarch, Wang Mang 王莽, by embodying the metrological standards for the new dynasty. This vessel is shaped like a large barrel with two ears. Overall the vessel contains five compartments, one for each of the five standard measures of weight, volume, and depth (which provides length). An eighty-one-character inscription glorifying Wang Mang's ascent to power is engraved on the main body of the vessel. Named by modern scholars after its designation in the inscription, the *Jia Liang Hu* provides some of the most important information on and the standardization on weights and measures in the imperial court of the first century. Because the inscription was made at the time of its casting, researchers are able to directly corroborate textual information with the physical weights and measures.[316] This is

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[315] Zheng Rongda 鄭榮達, "An Examination of the Value and Length of Huangzhong and the Measuring Ruler in the Western Han. *Xi Han Huangzhong biaocheng lü chang yu du chi Kao* 西漢黃鐘標稱律長與度尺考," *Huangzhong* (*Wuhan Music Conservatory*)《黃钟(武汉音乐学院学报)》, no. 1 (2017): 127–39.

[316] Loewe 2016, 145–46. At times, inscriptions were added to vessels even centuries after their casting. The *Jia Lliang Hu* allows us to rely on the script and compare it to the vessel, which is meant to embody the conditions presented by the text.

a rare, if not unique, occurrence, in which the textual requirements can be matched against an actual extant example meant to embody them. As we have seen above, the *Han shu* LLZ discusses such a vessel, and provides information on its weights and measures as well as its cosmological significance.

Although not excavated from a tomb, the *Jia Liang Hu* seems to have been passed down through the centuries, and currently resides in the National Palace Museum in Taipei. This separates it from excavated material, yet its dating has not been contested, and it is treated as an authentic relic.[317] The purpose of such a vessel was not for daily use, but to serve as an exemplary model, embodying the weights and measures of Wang Mang's Xin dynasty, "designed to be the standard for conformity and copying."[318] The comparison between this exemplary vessel and other contemporary daily measuring tools helps determine whether standardization was adhered beyond the confines of the court. The division of the vessel's compartments is as follows:

	Left Ear	**Main Vessel**	**Right Ear**
Upper	*Ge* 合	*Hu* 斛	*Sheng* 升
Lower	*Yue* 龠	*Dou* 斗	

That the vessel was transmitted rather than excavated can be reassuring to those who we suspect that burial items may not reflect real and practical instruments used by society. The measurements of this vessel allow comparison with excavated measuring tools from around the same period.

[317] On the problem of dating bronze vessels and inscriptions from the Qin and Han, see Loewe 2016, 153–54.

[318] Michael Loewe. *Problems of Han Administration: Ancestral Rites, Weights and Measures, and the Means of Protest* (Leiden ; Boston: Brill, 2016), p. 156

Figures 1: The *Jia Liang Hu*[319]

With the inscription facing the viewer, the *hu* upper section of the main vessel is one *chi* 尺 high, which stands at 23.1 cm. The lower *dou* 斗 section of the main vessel is one *cun* 寸 high, which stands at 2.3 cm. On the left ear, the upper part is a *ge* 合, and the bottom is a *yue* 龠. The *ge* is one *cun* high, and the *yue* 龠 is only 5 *fen* 分 high. In order to use the *dou* or *yue* receptacles, one must flip the vessel over.

[319] Xin Dynasty (9-23 C.E.), *Jialiang Hu* 嘉量斛, First Century, National Palace Museum in Taipei, www.npm.gov.tw/exh95/chinhan95/pic02_ja.html.

The eighty-one-character inscription on the outer section of the vessel discusses the celestial conditions and fateful date of Wang Mang's ascent to power; his standardization of weights and measures; and his setting of the calendar. It makes no mention of *Huangzhong*, and tuning standards only appear in reference to Wang Mang's grand act of standardization. The link to *Huangzhong* is made in one of five additional minor inscriptions on the body of the vessel, which I discuss and translate below. The main inscription is not exclusive to the *Jia Liang Hu*. It appears on at least two other objects dated to the Xin dynasty: a steelyard balance, and a bronze volume measuring vessel.[320] For context, I provide a translation of the main inscription, following Loewe's transcription and comments:

> 黃帝初祖，德帀於虞，虞帝始祖，德帀於新，歲在大梁，龍集戊辰，戊辰直定，天命有民，據土德受，正號即真，改正建丑，長壽隆崇，同律度量衡，稽當前人，龍在己巳，歲次實沈，初班天下，萬國永遵，子 孫，亨傳億年.
>
> Huangdi, was the founding ancestor; his virtue extended*[321] to Yu. When Yu di was the establishing ancestor, his virtue extended to Xin. The Year Star (Jupiter) was stationed at Da Liang; and the Dragon (Jupiter) was positioned on [the year] Wu Chen.[322] Wu Chen was directly fixed, and Heaven's Mandate took possession of the people. [He] received [the mandate] according to the virtue of earth*; corrected [his] title and took his true position. [He] recalibrated the calendar, establishing *chou* [as the first month]. Everlasting, grand, and sublime; he corresponded between the standards of tuning, length, volume, and weight; conforming with those before him. When the Dragon was positioned on *Jisi*, the station of the Year Star was *shishen*. When [his rule] first spread to all under heaven,

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[320] I make this judgment based on an image of the rubbings taken from these object. Both are indicated as excavated objects. The steelyard was excavated in 1937 in Gansu province. The bronze volume measure was excavated in Henan, in the late Qing dynasty. I did not make any further verifications on their authenticity. Images of the objects and a short analysis of each appears in *Guojia jiliang zongju* 國家計量總局, ed., *Zhongguo gudai du liang heng tu ji* 中國古代度量衡圖集 (Beijing: *Wenwu chubanshe* 文物出版社, 1981), 84-5,142-3.

[321] Characters or sentences marked with an asterisk are especially difficult. I follow Loewe's translations.

[322] The inscription refers to the twelve stations of Jupiter Cycle. This was Liu Xin's astronomical innovation, a short lived method, which re-defined the mean year as 1/12 of the sidereal motion of Jupiter, passing a station per year in a twelve year cycle. Sivin, "Cosmos and Computation," 15–16.

> the many states eternally complied. Sons and sons, grandsons and grandsons will enjoy [his] legacy for a myriad years.

The inscription credits Wang Mang's legitimacy to the virtuous powers of heaven and earth, who bestowed the mandate upon him. Wang Mang recalibrated and reset the guides of heaven, by adjusting the calendar, and earth, by adjusting tuning standards, weights and measures. He is presented as one who brings order to the whole world (*tian xia* 天下*, wan guo* 萬國). The tuning standards are mentioned as part of the earthly package in the quote taken from the *Book of Yu*, which also appears in the *Han shu* LLZ: "The *Book of Yu* says: 'Then, he made the standards of tuning, length, volume, and weight correspond'"(*nai tong du liang heng*《虞書》曰「乃同律度量衡」). The language of the main inscription certainly resembles that in Liu Xin's text. Loewe labels it a propaganda text, that may have also meant to separate Wang Mang from the Liu house of the Western Han.[323]

Each of the five minor inscriptions provides the formula for the calculation of the area and volume of each measure, using similar wording to the *Han shu* LLZ. Their nature is instructive and strikingly different from the assertion to rulership described in the main inscription. When Loewe compared different rubbings of the minor and main inscriptions, he noted that the calligraphy for the characters *lü* 律 and *Huang* 黃 is identical in the small inscriptions, but differs from the main inscription, which suggest that they may have been added to the vessel after the main inscription. This, however, remains uncertain.[324]

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[323] Loewe, *Problems of Han Administration*, 225, 227–32.

[324] If, indeed, the minor inscriptions are a later addition, Loewe suggests they probably were added sometime before the middle of the third century, when they would have been obsolete. If they are not independent, but draw on the *Han shu* treatise, Loewe suggests that they would have been added after the completion of the work, "some time after Ban Gu's death in 92 CE," Loewe, 226.

But how then does the *Jia Liang Hu* relate to sound? The smallest of the five compartments, called *yue* 龠/籥 (*lewk),[325] is a volume measure related, at least in the *Han shu* LLZ, to the *Huangzhong* pitch pipe.[326] One of the five minor inscriptions identifies the *yue* and the formula for its volume related to the *Huangzhong* pitch pipe. Before the *Han shu*, the *yue* is mentioned only as a wind instrument used in ritual performance. The *Shuowen jiezi* and the *Fengsu Tongyi* gloss *yue* it as an ordinary three-holed flute.[327] It is not the same as a pitch pipe, which had no tone holes and was open on both ends. So far, there is no evidence of a *yue* measure of volume in the Warring States and Qin periods.[328] One should keep in mind that measures prior to the Qin unification differed between states. States shared some of the nomenclature, at times due to conquest or trade.[329] A *yue* volume unit, said to be based on a *Huangzhong* pitch pipe, appears only from the Han onward. It is possible that it was created as part of a metrosophical statement in the Western Han, aimed to give credence to the link between sound and weights and measures, but we cannot be certain.

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325 Baxter and Sagart, *Old Chinese*, 374.

326 For an overall account and analysis of the remaining weights and measures of the *Jia Liang Hu* see Guangming 丘光明 Qiu et al., *Zhongguo Kexue Jishu Shi: Du Liang Heng Juan* 中國科學技術史：度量衡卷 (Beijing: *Kexue chubanshe* 科學出版社, 2001), 218–30.

327 The *Shuowen jiezi* glosses *yue* 龠 as the three holed flute, and 籥 as a bamboo mat used by yong students or attendands, which has no direct relation to music or to measurements. *The Fengsu tongyi* glosses *yue* 籥 as the three holed flute. Xu Shen 許愼, *Shuowen jiezi xin ding* 說文解字新訂, 130,291.

328 Loewe, *Problems of Han Administration*, 162. Ying Shao 應劭, *Fengsu tongyi jiaozhu* 風俗通義校注, 309.

329 Loewe, *Problems of Han Administration*, 156–58.

Figure 2: *The Jia Liang Hu*, left ear, top, *ge* 合

Figure 3: The *Jia Liang Hu*, left ear, bottom, *yue* 龠

The dimensions of the *Jia Liang Hu* and its various compartments have been measured and tested several times over the years, with slightly different results each time. Thus, results vary

for the diameter, depth, and volume of the *yue*.[330] According to measures published in 2011 by the National Museum in Taipei, the *yue* inner diameter is 3.3cm, its height is 1.2cm, its weight is 7.7295g, and its volume is 10.65ml.[331]

Rules of Precision: the area of a circle

A few additional *yue* vessels from the Xin dynasty also exist. One is known as a 2/3 *yue* (*Daban Yue Tong Liang* 大半籥銅量); its volume is 6.7ml, which does not match two-thirds of the *Jia Liang yue*. Another is a bronze *yue* (*tong yue* 銅籥) unearthed in 1970 in Xianyang 咸陽, Shaanxi province 陕西. It is in the shape of a ladle with a round top, flat bottom, and a long handle with a ring at its end.[332] The handle's façade is inscribes with the formula for deriving the area of a circle from a square, a process which required the use of Pi. It also includes the dimensions of the *yue*, and a statement that the volume of the *yue* derives from the *Huangzhong* pitch pipe. The date for the first year of Wang Mang's reign is inscribed on the underside of the handle. The published measurements are as follows: Inner diameter:

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[330] Qiu et al., *Du Liang Heng*, 218–19. Qiu presents various measurements of the *yue*, from the nineteenth century to the 1970′s. For differences, see, for example : In 1936, Li Naiji 勵乃驥 measured the diameter at 3.32 cm, depth 1.16 cm, volume at 10.04ml, while in 1928 Liu Fu 劉復 measured the diameter at 3.23 cm, depth at 1.29 cm, and volume at 10.57ml. In 2011, the National Palace Museum in Taipei published their results. I cite their version here. Loewe also relies on this source. See Meifen Cai, Chuanxin He, and Tianren Lin, eds., *Splendid Treasures: A Hundred Masterpieces of the National Palace Museum on Parade. Jingcai Yibai: Guobao Zongdongyuan Te Zhan.* 精采一百國寶總動員特展 (Taipei: Guoli Gugong Bowuyuan, 2011), 76–77.

[331] The National Museum does not specify volume measures in the 2011 publication. Qiu et al. provide the volume as 10.65 ml.

[332] For the 2/3 *yue* vessel dated to the Western Han see Loewe, *Problems of Han Administration*, 199. For the Bronze *Yue* ladle see Qiu et al., *Du Liang Heng*, 228.

3.164cm; outer diameter: 3.643cm; depth: 1.119cm; volume: 9.287ml.[333] I will discuss the inscription below.

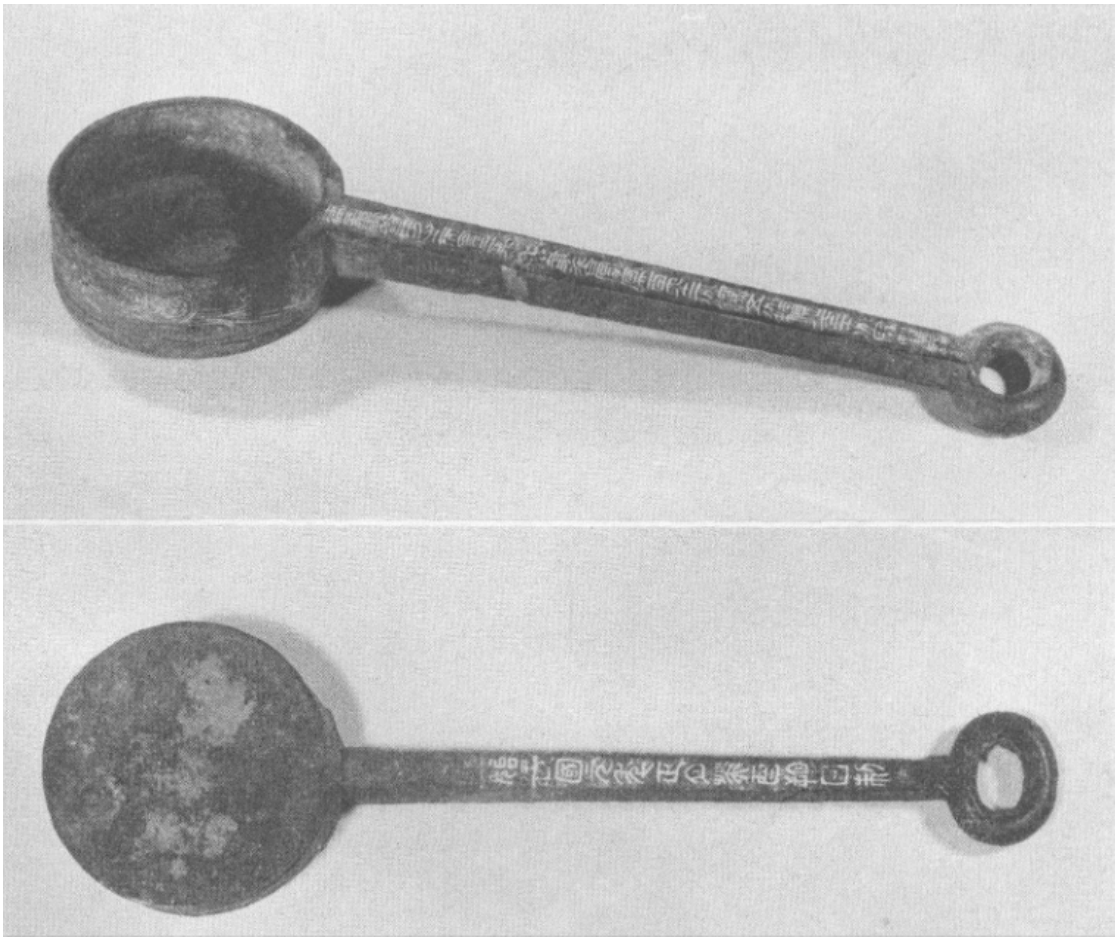

Figure 4: Bronze *yue* ladle 銅籥. Xianyang, Shaanxi. Xin dynasty.[334]

[333] Xianyang Museum 咸陽市博物館, *"Xianyang shi jinnian faxian de yi pi qin han yiwu* 咸陽市近年發現的一批秦漢遺物," *Kaogu* 考古, no. 3 (1973): 169.

[334] Xianyang Museum 咸陽市博物館, 207.

The inscription on the bronze *yue* 銅籥; the minor inscriptions for *the Jia Liang Hu* receptacles; and the *Han shu* LLZ all share a phrase depicting a circumscribed square (*fang chi er yuan qi wai* 方尺而圓其外). This phrase does not refer to the vessel's shape, or only describe its volume. It is, rather, an early formula by which to measure the diameter of a circle. According to Guan Zengjian 關增建, early Chinese literati had not yet thought of a way to accurately determine the diameter of a circle without using the dimensions of a circumscribed square.[335] The same method is also mentioned in the *Zhoubi Suan Jing* 周髀算經, although Cullen suggests that it may have been added to the text around the first century.[336]

The inscription on the bronze *yue* ladle reads:

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335 Guan deems Liu Xin's ability to use the *"tiao pang"* system as quite extraordinary, since he used a process which required the use of Pi π to deduce the diameter from a given area. (based on the *Jia Liang Hu,* the value of Pi value was 3.1547). Liu Xin was aware that simply calculating the area of the circle according to the sides of a circumscribed square (equaling 1 *chi* in the case of the *Hu*) produces an inaccurate result (157 cun^2) that does not meet the given requirement of 162 squared *cun*. The result is 5 cun^2 short. By adding 9 *li* and 5 *hao* to each end of the diagonal of the square and recalculating with the new diagonal, the result is correct. This is the Empty Space (*Tiao pang* 旁) method. I wrote out the calculations below using Liu Xin's value for π (the diagonal of a square is calculated by multiplying the side of a square by the root of 2):

- Without the *tiao pang* addition we calculate πr^2, where $r = (1 \times \sqrt{2}) \div 2 = 0.70710678118$. The area is then 1.57 chi^2 or 157 cun^2.
- With the addition of 0.0095 to each end of the diagonal, $r = \left(\left(1 \times \sqrt{2}\right) + 0.019\right) \div 2 = 0.71660678118$. The area then becomes about 1.62 chi^2 or near 162 cun^2

Other scholars have previously explained Liu Xin's value of Pi. See, for example, Bai Shangshu 白尚恕, "Cong Wang Mang Liang Qi Dao Liu Xin Yuan Lü 從王莽量器到劉歆圓率," *Beijing Shifan Daxue Xuebao* 北京師範大學學報, no. 2 (1982): 75–79.

336 Guan p. 129, Cullen 1996, 155, 172-75, 181–82. Cullen's translation of the full sentence reads: "The patterns for these numbers come from the circle and the square. The circle comes from the square, the square comes from the trysquare, and the trysquare comes from [the fact that] nine nines are eighty-one." (p. 174) 數之法，出於圓方。圓出於方，方出於矩。矩出於九九八十一.

律量籥，方寸而圜其外，庞旁九豪，冥百六十二分，深五分，积八百一十分，容如黃鐘。[337]

When tuning the volume of the *yue*, [use] a circumscribed square with sides measuring 1 *cun*.[338] The empty space is 9 *hao*; its volume is 162 [squared] *fen*; its depth is 5 *fen*; its area is 810 *fen*. Its capacity is like that of *Huangzhong*.

The inscription for the *Jia Liang Hu yue* receptacle reads:

律嘉量籥，方寸而圓其外，庞旁九釐五毫. 冥百六十二分，深五分，積八百一十分，如黃鐘容

When tuning the *yue* of the excellent measures, [use] a circumscribed square with sides measuring 1 *cun*. The unfilled space is 9 *li* and 5 *hao*; its volume is 162 [square] *fen*; its depth is 5 *fen*; its area[339] is 810 *fen*, like the volume of *Huangzhong*.

The formula in the *Han shu* LLZ:

其法用銅，方尺而圜其外，旁有庞焉。

For its method, use bronze. A circumscribed square with sides measuring 1 *chi*. The sides have an empty space between them (and the circle).

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[337] I follow Guan's reading of *ming* 冥 as *mi* 冪, meaning "surface," and referring to the area of the circle. Guan Zengjian 關增建, "Exploration of Liu Xin's Metrological Theory," 129.; Loewe, *Problems of Han Administration*, 233.

For the inscription transcription see: Xianyang Museum 咸陽市博物館, "jinnian faxian de yi pi qin han yiwu," 169. and Qiu et al., *Du Liang Heng*, 228.The original inscription seems to mention a *tiao pang* of only 9 *hao*. Adding 9 *hao* to both ends of the diagonal of a square with sides measuring 1 *fen* produces an area which is very close to 162 fen^2. It is close enough to the *Jia Liang Hu* inscription to suggest a mistake in copying, or a recognition that calculating beyond the *hao* 豪 was either too difficult or unnecessary. However, the measured volume of this bronze *yue ladle* is 9.287ml, while none of the measures of the *Jia Liang yue* volume came to less the 10 ml (see p. 147). Since both are inscribed with the same reign year, and the ladle was discovered in Xianyang, in the vicinity of the Xin capital, Chang'an, it points to inaccuracy in copying. Additionally, it was not excavated from a tomb, but discovered in a field during the hoeing season, so we have no way of assigning it to an owner.

[338] I chose to translate *lü* 律 as "tuning" here, because of its mechanical sense and acoustic undertone, which seem to fit its function in the *Jia Liang Hu*. The is clear by the reference to *Huangzhong* both on the inscription of the *yue* measure and in the *Han shu LLZ*.

[339] *Ji* 積 here refers to the overall volume of *yue*.

Tiao Pang 朓旁 refers to the method of calculating the empty space between each corner of the square and the circle.[340] Liu Xin used the method to calculate the area of a circle from a square, in order to determine its diameter.[341] The formula appears in inscriptions on measuring vessels probably to assist other manufacturers to copy accurately. Another likely reason to assert knowledge of heaven and earth by deducing measures from the circle and the square.[342] Vogel analyzes the *Han shu* LLZ description, saying that "The individual volume measures were round both inside and outside, but were conceived to contain imaginary squares inscribed into the circles of the container."[343] We now know that these squares were not only a literary cosmological metaphor, but were used for practical purposes in addition to ideological and rhetorical ones. Wang Mang's vessel, Liu Xin's formula, and his *Han shu* section on weights and measures are a testament to the gravity they assigned to precision as part of the imperial rhetoric.

Rulers and Rulers: Foot-rules and the length of the *Huangzhong* pipe.

Analyzing measuring tools, as well as other artifacts that may include measure indications in writing (e.g. lamps, coffins, musical instruments), or that we know their decreed measure (e.g. official coins), allow us to determine the state's standard, and how closely people inside and outside of the court obeyed it. Foot-rules are slats made of bone, wood, or bronze, used

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[340] Loewe follows Li Naiji, who suggests the character *tiao* in the inscription is possibly a *ci* 庇. Loewe, *Problems of Han Administration*, 225–26.

[341] Guan Zengjian 關增建, "Exploration of Liu Xin's Metrological Theory," 129.

[342] Vogel brings up this point with regards to the description of the vessel in the *Han shu* LLZ.

[343] Vogel, "Aspects of Metrosophy and Metrology during the Han Period," 138.

to measure length. The overall length of a foot-rule was usually a *chi* 尺 or a *zhang* 丈, marked with smaller units of measure, such as *fen* 分 or smaller.[344] The *Han shu* LLZ ties lengths to the foot-rule and to the derivation of the length standard from the *Huangzhong* pitch pipe. But excavated foot-rules pose several problems for scholars, among them are the natural erasure of their divisions over time, bending, cracking, and breaking of the slats, and uncertainty about whether it was used as a standardizer or for daily tasks.[345] Until recently, conclusions were based on artifacts excavated until the mid 1990's. By now, the number of excavated measuring tools from the Warring States, Western Han, Eastern Han, and Jin tombs has increased exponentially, and scholars are continuously refining our understanding of the evolution of weights and measures.

In 2015, Bai Yunxiang published a short survey of foot-rules dated to the Western Han, Xin, and Eastern Han periods, excavated over the past two decades.[346] The new data confirms that the standard length of a *chi* during the Western Han and Xin dynasties was 23.1cm, but the variation has increased greatly. It is now clear the 23.1cm is only a "relative general value; in the practices of archeological studies, we should [not] absolutize it."[347] The data also show a low consistency outside the court. The standard value was not widespread

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[344] The *Han shu* LLZ begins with *fen* as the smallest length measure, but smaller units such as *li* 釐 and *hao* 豪 also existed.

[345] Loewe, *Problems of Han Administration*, 187.

[346] Bai mentions bronze, wood, bone, gold inlaid iron, jade, ivory, and bamboo as the variety of materials in Han dynasty rulers. Of these, bone, wood, and bamboo were most popular in the Western Han, and bronze and bone were most popular in the Eastern Han. The Xin has only two examples so far, both of bone. Yunxiang Bai, "The Studies on the Measuring Devices of the Han Dynasty and the Relevant Issues," *Chinese Archaeology* 15, no. 1 (January 1, 2015): 191.

Bai Yunxiang is the deputy director of the Institute of Archaeology, Chinese Academy of Social Sciences, and deputy chief editor of *Kaogu* and Chinese Archaeology.

[347] The changes are my own. The article is translated from Chinese. It says: "avoid to absolutize it. "Bai, 193.

across time or space. The length of a *chi* in the Western Han ranged between 21.7cm and 23.8cm, and the two foot-rules dated to the Xin dynasty measured 22.9cm and 23.1cm respectively. Those of the Eastern Han were longer, ranging between 22.9cm and 24.1cm.[348] Bai also concludes that there are sharp differences between measures taken from material objects, which were closer to the standard value, and those taken from the foot-rules, which varied greatly, as seen above.[349] It seems that the differences arose not from an intentional disregard for the central standard, but due to variations in the length of the *chi* in each location at certain points in time, and by different craftsmen.[350] An additional reason could be the different uses of different tools. Some tools may have required a greater level of precision than others.

As for the usage of the foot-rules, one of the most striking discoveries this survey provides is the gender of the identifiable tomb occupants. Out of 78 foot-rules excavated from Han tombs, eleven were found in tombs whose gender was identifiable. Out of the eleven, nine were women and two were male. Bai concludes that the foot-rules were used in the daily lives of the tomb occupants and are thus reliable for the study of practical measuring instruments.[351]

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[348] According to Zheng Rongda, the *chi* was 23.1cm, but it is not uncommon to find discrepancies of about 5 mm in the measurements of a *cun*. Zheng Rongda 鄭榮達, "An examination of the Value and Length of Huangzhong," 132. Zheng relies on material which does not postdate the 1990′s. Bai′s report shows much greater variance. Loewe argued that the length of a *chi* between the Western and Eastern Han ranges between 22.3 and 23.4 cm. Loewe, *Problems of Han Administration*, 186.

[349] Bai, "The Studies on the Measuring Devices of the Han Dynasty and the Relevant Issues," 191,193.

[350] Bai urges scholars to calculate lengths with an eye towards the Han perspective instead of only relying on the modern values. Looking for the Han unit could facilitate the calculation of the real *chi* value as the people of the particular time and place used it. He gives the example of the "T-shaped silk funeral banner unearthed from Mawangdui Tomb No.1 in Changsha, [which] is 205 cm in length; if we measure it with 23.1 cm as 1 *chi* of the Western Han dynasty, its length is counted as 8.87 *chi*; however, if we measure it with 22.8 cm as 1 *chi*, its length will exactly be 9 *chi*." Bai, 191.

[351] Bai, 192–93.

The standard of 23.1cm began earlier than the Qin and Han, in the late Warring States period. This can be seen in two examples: a bronze foot-rule of the Eastern Zhou state excavated from the *Jin Cun* mound 金村 mound in Luo Yang, Henan county, in which a *cun* equals 2.31cm. The foot-rule possibly belonged to the ruler of the state, making it more plausible to have been standard. Its inner divisions, however, are not uniform. It is divided into ten uneven sections, one of which has eleven sub-divisions.[352] The Museum of Chinese History[353] holds two bronze steelyards, belonging to a weighing apparatus, each 23.1cm long. The samples are dated to the Warring States period, and were excavated in Anhui, the territory of the Chu state.[354]

There is no dispute among scholars regarding the common variance in weights and measures outside the central court. It does not exist solely in foot-rules and volume measures, but also in other objects such as the Xin-dynasty *Huoquan* 貨泉 and *Huobu* 貨布 coins. The standard diameter of the *Huoquan* coin was supposed to measure one *cun*, and so did the bottom of the *Huobu* coin. Many of excavated coins exhibited variance from the state standard.[355] This variance could have been caused by a number of reasons ranging from lack of oversight, inaccurate copying methods, illegal minting, corruption, and poor communication with the central court. The Qin legal documents excavated from Tomb no. 11 at Shuihudi 睡虎地 (217 B.C.E.), although belonging to the Qin dynasty attest to early imperial attempts at controlling weights and measures. The government implemented fines

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[352] Qiu et al., *Du Liang Heng*, 156.; Zheng Rongda 鄭榮達, "An examination of the Value and Length of Huangzhong," 137.

[353] Today part of the National Museum of China

[354] Qiu et al., *Du Liang Heng*, 134.

[355] Zheng Rongda 鄭榮達, "An examination of the Value and Length of Huangzhong," 131.

for inaccurate measures and failure to regulate; annually inspected prefectural and state workshops; and lent out reliable models. For example, according to the "Miscellanies of the Governor of the Capital Area" (*neishi za* 內史雜), officials and prefecture staff (*xian liao* 縣料) regulated weights and measures, and it was forbidden to lend models to commoners. Standardized measuring vessels were to be used only within the government.[356] Nevertheless, the statutes evidence intention, nut not implementation. As Li concludes: "The Shuihudi documents indicate legal principles while their implementation might have been different."[357]

Clearly, we cannot ascribe any attempts at oversight made by the Qin dynasty to the Western Han or the Xin. The "Statutes on Cash" (*qian lü* 錢律) may be the only surviving example from the early Han to show an administerive attempt to "rein in the chaos created by a completely unregulated free currency" due to Liu Bang's precarious power and lack of control over resources during the first years of the Han.[358] However, these statutes touch upon the regulation of coin size only very briefly and are much more concerned with penalties for private minting, counterfeiting, and melting official coins to make bronze

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[356] Kin Sum (Sammy) Li, "To Rule by Manufacture: Measurement Regulation and Metal Weight Production in the Qin Empire," *T'oung Pao* 103, no. 1–3 (August 28, 2017): 28.Li provides a new translation of the Shuihudi statutes concerning the Qin regulation of weights and measures. For a short summary see Loewe, *Problems of Han Administration*, 159–60.

[357] Li, "To Rule by Manufacture," 29.

[358] These legislations were presented during the reign of Empress Lü (r. 194-188 B.C.E. as regent, and 187-180 B.C.E.), and were removed and reinstated until the reign of emperor Wu (r. 141 –87 B.C.E.). Anthony J. Barbieri-Low and Robin D. S. Yates, *Law, State, and Society in Early Imperial China: A Study with Critical Edition and Translation of the Legal Texts from Zhangjiashan Tomb No. 247*, vol. 2, (Leiden ; Boston: Brill, 2015), 627-628.

vessels.[359] Currently, we have no definite evidence like the Qin legal text to show attempts at territorial oversight for weights and measures or lack thereof during the period of the Western Han and the Xin. It is possible that these two courts were sufficiently pleased with standardization at court or in the capital, although I find it unlikely. Nevertheless, both cases argue for a fantasy of control. If the variance in measurement occurred despite court supervision, we can argue for a de facto lack of control. If the variance in measurement occurred due to a very centralized, ritual metrosophy, we can argue for official rhetoric that promoted the idea of an all controlling emperor.

Black Millet and the *Huangzhong* Pitch pipe

From the Han dynasty onward, the *Huangzhong* pitch pipe, the lowest pitch in the cycle of tuning standards, the longest of the string and pipe lengths, was predominant. It began the pitch cycle and was the origin for the calculating the rest of the tuning standards. Cosmologically, it was associated with the eleventh (civil) month, and the rebirth of the *yang* force at the winter solstice, which commenced the astronomical year. Mythological accounts, such as the tale of Ling Lun, and the *Han shu* LLZ tell the reader that the measurements are meant for a pipe rather than a string.[360] The *Han shu* LLZ set *Huangzhong* at 9 *cun* for the

[359] For the translation of this portion of the statutes see: Anthony J. Barbieri-Low and Robin D. S. Yates, *Law, State, and Society in Early Imperial China: A Study with Critical Edition and Translation of the Legal Texts from Zhangjiashan Tomb No. 247*, vol. 2, Sinica Leidensia (Leiden ; Boston: Brill, 2015), 632-633.

[360] The *Lüshi Chunqiu* version of this myth sets the length of the *Huangzhong* bamboo pipe at 3 *cun* 9 *fen* (其長三寸九分而吹之，以為黃鐘之宮). The *Shuoyuan* version sets it at 9 *cun* (其長九寸而吹之，以為黃鐘之宮), the *Han shu* LLZ version does not mention its size. Elsewhere in the treatise *Huangzhong* is 9 *cun*.

reign of Wang Mang.[361] Liu Xin defines each of the standards also by an amount of millet grains required to measure it. According to Qiu Guangming 丘光明, the use of millet grains for measuring was meant to reconcile abstract theory with concrete measurements, providing an easier, more feasible, method of measuring, rather than having to handle diameters and other complex calculations for volume and weight. Based on the length of a *chi* unit as 23.1cm, and the diameters of the *yue* from the *Jia Liang Hu*, Qiu's calculations led her to argue that, in theory, a *Huangzhong* pitch pipe during the reign of Wang Mang would have been set at a frequency of 384.8_{Hz}– a bit lower than a G^4 pitch today.[362] This is a fascinating theoretical experiment, but results that do not derive from an actual pitch pipe unfortunately cannot be verified. Converting pitch pipe measurement based on the *Han shu* LLZ criteria, especially using millet grains, is problematic in several ways, and has been a point of dispute for centuries; as Gao refers to it as "the eternal dispute".[363]

One point of the dispute regarded determining the length and pitch of *Huangzhong*. Did sound determine the measures, or did measures determine the sound? Several early literati argued that the sound had to be contained in material objects in order to transmit its measures to future generations. During the Northern Song (960-1127), Fan Zhen 范鎮 (1007－1088) lamented the disorder of his own time, and harkened back Liu Xin's requirements. He begins with a discussion on harmonized *qi* and the nature of sound,

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361 The *Shiji* "Book on Tuning Standards" is confusing, because it provides two options for *Huangzhong*: at 8.71 *cun* (黃鐘長八寸七分一，宮), as well as 9 *cun* (凡得九寸，命曰「黃鐘之宮」。).

362 Qiu bases her calculations on the use of millet, following the *Han shu* instructions, and comparing them to known measurements from excavated material. Qiu et al., *Du Liang Heng*, 46.

363 Gao Zhensheng 高振聲, "*Han shu leishu zhi zheng xin tan*《漢書》累黍之爭新探," *Nongye Kaogu* 農業考 古 1 (2016): 35.

arguing that vessels, which have shape, helped transmit information about the nature of sound, which is otherwise abstract and intangible:

> 發和氣者，聲音也。聲音之生，生於無形，故古人以有形之物傳其法，俾後人參考，然後無形之聲音得而和氣可導也。
>
> Sound means issuing forth harmonized *qi*. In generating sound, it is generated from the formless. Thus, the ancients used objects which had form to transmit its model, enabling later generations to verify and examine [it]. Then, the formless sound materializes, and harmonized *qi* can guide [them].[364]

He enumerated ten tools, eight of which are itemized in the *Han shu*, such that measures would be deemed correct only when they correspond with each other: millet grains, tuning standards, foot-rules, the *yue*, *ge,* and *hu* [volume] vessels, arithmetic 算數, weights and balances, bells, and chimes. Not much is said about how this correspondence should be accomplished. The addition of bells and chimes is probably meant to supply the musical connotation, having the fixed pitch instruments answer the determined *Huangzhong* pitch.

In the Ming, renowned acoustician Zhu Zaiyu 朱載堉 (1536 – 1611)[365] repeatedly negated the claim that pitch was determined by foot-rules and not the other way around. He argued that this is an erroneous approach to the conceptualization of sound as natural and as dictating measures rather than shaped by them. This was part of a larger criticism Zhu Zaiyu

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[364] The *Wenxian Tonkao* 文獻通考 is a Yuan dynasty encyclopedia compiled by Southern Song literati Ma Duanlin 馬端臨. The section on "the Generation of Tuning Standards in Successive Generations" 歷代製造律呂(in 卷 131, "Examining Music 4"樂考四) is a gargantuan chronological collection containing intricate discussions on sound, weights and measures, and the use of millet grains. Ma Duanlin 馬端臨, "Wenxian tongkao 文獻通考," in *Siku quanshu* 四庫全書, vol. 610–616 (Shanghai: Shanghai guji chubanshe, 1987), 613: 68-69.

[365] Zhu Zaiyu is revered as one of the most prolific acousticians in pre-modern China. He is well known for constructing the formula for equal temperament.

had toward Liu Xin's measure of *Huangzhong* as 9 *cun*.[366] He wondered why Liu Xin set the measure of *Huangzhong* and added 1 *cun* to create the foot-rule. If measures derived from sound, this should not be.[367] He also argued that it is the nature of the thickness of bamboo pipes to differ on both ends; that each inner diameter 孔內 differs from the others; and that since the *Huangzhong* pitch pipe was meant to come from nature, if man meddled with its natural characteristics, emptied it, evened out the walls and the ends, it would no longer fulfil its original purpose. "How could he [thus] derive a true number?!" (*an neng de qi zhen shu ye zai* 安能得其真數也哉).[368]

On the same problem, Dai Nianzu 戴念祖, a modern historian of physics, has raised the problem of the effects of temperature and moisture on the velocity of sound, as well as the angle of the lips on the pipes, which could affect the pitch.[369] The modern understanding of the behavior of soundwaves makes clear the problematic aspects of using natural sound as a standard for measure, because a host of variables other than the length of a pipe should be taken into account.[370] Nevertheless, these issues are not raised in any early

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366 Zheng Rongda analyzes this dispute in depth, but much of it is strictly theoretical. Zheng Rongda 鄭榮達, "An examination of the Value and Length of Huangzhong."

367 Zhu Zaiyu 朱載堉, *Lulu jingyi* 律吕精義, ed. Fang Wenci 冯文慈 (Beijing: Renmin Yinyue 人民音乐, 2006), 5. For additional examples to these ongoing disputes see Qiu et al., *Du Liang Heng*, 47–48.

368 Zhu Zaiyu 朱載堉, "Lüxue xinshuo 律學新說," in *Siku Quan Shu* 四庫全書, vol. 213 (Shanghai: Shanghai gu ji chu ban she, 1987), 636.

369 Dai Nianzu 戴念祖, *Zhu Zaiyu, Mingdai de kexue he yishu juxing* 朱載堉, 明代的科學和藝術巨星 (Beijing: Renmin chubanshe 人民出版社, 1986), 206.

370 Measuring sound using strings would have also been successful and would have eliminated many of the concerns Dai Nianzu raises. The problem would then become recreating the pitch on a pipe, which could be adjusted by ear.

Chinese texts, and we have no indication of the concerns of musicians or those playing the pitch pipes until the Western Jin dynasty (晉 265-316 C.E.).[371]

Over the years, scholars chose two ways to infer the size of *Huangzhong* from the LLZ statement on measuring the dimensions of the pitch pipe. One method was the "Pile by Single Grain Increments" (*Lei shu* 累黍) method, and the second relied on mathematical and geometrical calculations. As shown above, these were based on the *Jia Liang Hu* inscriptions, the formula for calculating the diameter of a circle, and a careful examination of the excavated material dated to Wang Mang's period.

Anyone who attempts to reconstruct the millet method faces several problems. The first is choosing the correct species of grain. Which species of millet was Liu Xin referring to? The specifications in the texts are unclear, and the meaning of *zi gu ju shu* 子穀秬黍 remains uncertain. Another problem is how to interpret *zhong zhe* 中者, which characterizes the grain. The correct application of the millet method became another point of dispute, which lasted centuries. Below, I provide a long, yet partial, list of some of the problems that arose from antiquity to modernity in the search for the millet grain method.

In the commentary to the *Han shu* LLZ, Meng Kang 孟康 (third century) argued that one should use Northern black millet, while Yan Shigu 顏師古 (581–645) disagreed, arguing that the locus of origin is of no consequence. Yan Shigu also interpreted *zhong zhe* as

[371] For a detailed account on the acoustic and aesthetic theory of Xun Xu (d.289) and his project to re-standardize tuning standards and weights and measures see Howard L Goodman, *Xun Xu and the Politics of Precision in Third-Century AD China* (Leiden; Boston: Brill, 2010).

referring to medium size grains (*bu da bu xiao* 不大不小).[372] Zhu Zaiyu disagreed and argued that *zhong zhe* referred to usable grains (*gai wei jianxuan zhongyong zhi shu*, *fei wei zhonghao* 蓋謂揀選中用之黍, 非謂中號), and that the character *ju* 秬 already referred to a large-sized grain, and so grains should be of top quality and large size.[373] The *Book of Wei* 魏書 also recounts the continued confusion surrounding Liu Xin's measuring units and the millet grain system three hundred years later. As the court attempted to recalibrate their tuning standards and the calendar, Gongsun Chong 公孫崇 constructed a new measuring ruler (*chi* 尺), which he claimed was based on millet grains. The Minister of Ceremonies, Liu Fang 刘芳 (453-513 C.E.) ordered the music to be corrected according to it, but Commandant-in-Ordinary Yuan Kang 中尉元匡 denied the results. For a long time "the three could not reach an agreement" (*sanjia fenjing* 三家紛競), until the emperor intervened to set the measures.[374]

The *Book of Sui* LLZ refers to a case in the Northern Zhou (557-581 C.E.) in which Emperor Xuan 宣 (559–580 C.E.) sent officials to search and bring black, round millet grains originating on Mount Yangtou of Shangdang 上當羊頭山.[375] The text tells us that because of a drought the grains differed in size. When using large grains, they measured well

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372 Ban Gu 班固, *Han shu* 漢書, 4:967. Meng Kang argued that *zi* 子 referred to the north, and so this refers to black millet originating in the north (*zi bei fang, beifang hei, wei heiju ye* 子北方，北方黑，謂黑黍也). Yan Shigu denied this and argued that *zi* simply referred to millet, and 秬 *ju* to black millet.

373 Zhu Zaiyu 朱載堉, *Lulu jingyi* 律吕精義, 798.

374 *Wei shu* 魏書 (Beijing: Zhonghua shuju 中華書局, 1974), 2657–58. Qiu et al., *Du Liang Heng*, 49. Gao Zhensheng 高振聲, *"Hanshu leishu,"* 35. For additional examples from premodern history see Gao Zhensheng 高振聲, *"Hanshu leishu."*

375 Wei Zheng 魏徵 et al., *Sui shu* 隋書 (Beijing: Zhonghua shuju 中華書局, 1982), 406–7.

with the requirements in the *Han shu* LLZ – one hundred grains per *chi*. But twelve hundred did not fit in a *yue* volume. Thus, the size, shape, and side (vertical or horizontal) of the grain continued to be a problem right up to the end of imperial China.[376] According to Qiu, mistaken identification continued, and the Qing scholar Wu Dazheng 吴大徵 (1835-1902), mistook *ju shu* 秬黍 for broomcorn (*gaoliang mi* 高粱米) – a mistake which carried over to Japanese scholarship on weights and measures.[377]

The "eternal dispute" kept questioning the appropriate time of year for the sowing, and the vertical or horizontal placement of the grains when aligning them side by side and tried to fill the gaps in information left by the *Han shu* requirements. Reconstructing from text alone, in other words, was an impossible task. Due to the wide variety of millet grown in China, and the lack of material evidence for pitch pipes, millet, and measuring vessels, it remained impossible to know whether and how this method was indeed practiced during the Han or Xin dynasties. In the late sixteenth century, Zhu Zaiyu went as far as to grow millet grains of his own.

Ancient artifacts such as the *Jia Liang Hu* and excavated implements came to the aid of modern scholars, such as Wan Guoding 萬國鼎 (1958), Zhao Xiaojun 趙曉軍 (2010) and Gao Zhensheng 高振聲 (2016). Once material evidence became available, the problem no longer hinged on determining the Xin measures, but to find a way to get length, volume, and weight to conform with each other accurately.

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376 Shangdang is a location in the central plains. In the *Discourses of States* it is referred to as a state 上黨之國.

377 Qiu et al., *Du Liang Heng*, 48.

Qiu surveyed red, yellow, and black millet grain varieties, separated each type into groups of large, medium, and small grains, and measured each group as instructed in the *Han shu* LLZ: 1200 grains to measure weight and volume, and 100 grains to measure length. She then compared the results against the *Jia Liang Hu* 12 *zhu* 銖 weight (7.4g), the *yue* volume (10ml)[378], and the *chi* length (23.1cm). None of the results came up accurate on all three counts, but also none were wildly inaccurate, pointing to a good possibility that his method was indeed used in practice.[379]

Qiu Long 邱隆 attempted a reverse process. He calculated the measures of a *Huangzhong* pitch pipe according to the available criteria and constructed a pipe made of glass.[380] Using millet from the Hebei area, his results matched 100 grains, set in line reached 23cm (not 23.1), and the volume could fit between 960 to 970 grains. The weights of the grains ranged between 6.9-7g. When Qiu Guangming compared her own results to Qiu Long's pitch pipe, and another early experiment by Wan Guowei 萬國鼎, she lamented the overall discord among grain types, methods used for measuring, and scholarly approaches. However, she recognized that medium-sized grains produced good results on average: 100 grains lined up equaled 23cm, 1200 grains weighed between 7.4 g and 8.5g, and a *jin* 斤 weighed between 224 and 256g. She argued reasonably for possible small changes in weights

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[378] Liu Fu 刘复 and others measured the *yue* as 10ml for the Western Han and 10.65ml for the Xin dynasty. Qiu et al., 219.This is problematic for the millet grain method because it would imply a change in the *Huangzhong* pitch pipe, if we accept the notion that the millet method was carried out, or that the *yue* is a product of the pipe. The *Jin* 斤 weight remained the same between the Western Han and the Xin, at 250g.

[379] For example, the Red Millet 紅黍 medium sized grains were accurate in weight and length, but not in volume. The medium grains of each group tended to produce results closer to the vessel measurements.

[380] Length (*shen* 深) = 20 cm; diameter = 0.8cm; Volume = 10ml.

and measures in the 400 years between the Western and Eastern Han, so that the average result still seems to be in the correct vicinity of an average measure.

As for Liu Xin, Qiu was certain that he deliberately formed the bond between sound, grains, and weights and measures, with the aim of turning the intangible nature of sound into the tangible, comprehensible, and reproducible. The problem that Qiu was unable to resolve was how to produce a result that would answer Liu Xin's requirements in every respect. How could 1200 grains fill a volume of one *yue* while weighing 12 *zhu* 銖, as the text requires. In her view, the limitations of science and technology in early China made it impossible, no matter which method was used, to meet the exact specifications.[381]

In 2016, Gao claimed to have finally found the solution. His main concern was that, over time, domestication made great changes to the original characteristics of black millet, and modern scholars who used it against the *Han shu* should have taken this into account.[382] For three years, in a plot of his own, he grew black millet from the Hebei area, the area of Mount Yangtou. He grew the millet under natural conditions, without the use of a chemical fertilizer or hormones and used the product of the seeds of the third year as a specimen. The conditions caused the grains to become smaller and weigh less than cultivated grains. He named this the Feralization Method (*yehua fa* 野化法). His results are surprising: the volume of 1200 grains was 10.7ml – extremely close to the 10.65ml *yue* volume of the *Jia Liang Hu*.[383] When piled in a row, they reached 23cm, 1mm short of the 23.1cm per *chi* in the Western

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381 Qiu et al., *Du Liang Heng*, 50.

382 Gao Zhensheng 高振聲, *"Hanshu leishu,"* 38.

383 Gao emphasized that he repeated the experiment several times, and the results were unchanged.

Han and the Xin.[384] In weight, 1200 grains weighed 8.01g, which differs by 3g from the Western Han and Xin measure of 12 *zhu* 銖, 7.8g. All discrepancies do not exceed 5%.

The results of the millet-grain experiments, especially those obtained by Gao, are close enough to the textual requirements that current scholars should not disregard it as mere rhetoric. We can conclude that Liu Xin constructed a system that used millet grains as the connector between sound and metrology, aiming to provide material proof to the conceptualization of sound as the nexus between man and the cosmos, with the emperor, Wang Mang, governing over it. Whether this system was used repeatedly, or anywhere outside the court remains unknown.

Finally, in 2011, in Nanchang city, Xinjian district, Jiangxi Province in south China, archeologists unearthed a very large tomb, richly furnished with over a thousand artifacts, among them many musical instruments, including a large bell set, chime stones, string instruments, and two cylindrical pipes made of jade. The tomb also contained large amounts of black millet. The tomb occupant was Liu He 劉賀 (r.74 B.C.E.) – a member of the ruling Liu family of the Western Han, who ruled China for all of 27 days, only to be deposed and demoted. His new title was the Marquess of Haihun 海昏侯. His identity was confirmed by a seal bearing his name. As of today, the analysis of the excavated material from Liu He's tomb is an ongoing effort, and it is considered one of the best-preserved and most important archeological discoveries from the Western Han.[385]

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[384] Gao Zhensheng 高振聲, "*Hanshu leishu*," 37.

[385] Yang Jun 楊軍 and Xu Changqing 徐長青, "Nanchang Shi Xi Han Haihun Hou Mu 南昌市西汉海昏侯墓," *Kaogu* 考古 7 (2016): 45–62.

Measuring and obtaining information from these grains as well as from the formal measuring tools and writings on objects unearthed in the tomb would be extremely useful for solving the mystery of the black millet grain method, and settling the "eternal dispute." Comparing these grains with Gaos' feral millet would also yield interesting results.

Figure 5: Black millet unearthed from the tomb of Liu He of the Western Han.[386]

386 This photograph is taken from an online resource. Wang Jinzhong argues that the two jade tubes (20cm and 18cm) found near the coffin are pitch pipes, and that as such are also standardizers of weights and measures of the Western Han. Because there is currently no way to examine these pipes I chose not to discuss them until more information is available. In the tomb's excavation report they appear as "jade cylinders" 玉管. Wang Jinzhong 王金中, "Yuzhi Huangzhong Lüguan: Haihun Hou Mu Faxian Handai Lü, Du, Liang, Heng de Jizhun Qi 玉質黃鍾律管：海昏侯墓發現漢代律、度、量、衡的基準器," Zhonguo Kaogu 中國考古, February 7, 2017, http://www.kaogu.cn/cn/kaoguyuandi/kaogusuibi/2017/0207/57014.html.

Conclusion: Between Theory and Practice

From the third century B.C, Chinese textual sources make sound a foundational concept and a tool for good rulership. The *Han shu* LLZ elaborated this rhetorical link through tuning standards, weights and measures, and mathematical astronomy – linking them through computation and cosmology. Archeological vessels, such as the *Jia Liang Hu*, the bronze *yue* ladle, and foot-rules, provide us with practical dimensions of this relationship between power and sound. A mainstay of the Emperor's claim to power was sound, especially that of *Huangzhong*: Upward, sound was a temporal and spatial cosmic vibration (as shown in Part Two of this dissertation). Downward, toward the people, sound was the origin of standardized measures, a display of imperial power and an indirect influence on the daily lives of the populace. As Nathan Sivin has concluded:

> It is a matter of historical record that cosmology had much to offer both political theory and administrative practice — which after all is why room was found for treatises on judicial astrology and mathematical astronomy in the Standard Histories.[387]

In theory, the display of a ruler's power began with an act of standardization, or a declaration of one. Liu Xin's guarantee of exact results, his confident assertion that "none fail to use it," and his demand for precision by providing the calculating diameters down to fourth decimal places, all assert vast and great control. The casting of an exemplary model like the *Jia Liang Hu*, with its formal inscription, which celebrates the ubiquitous acceptance of Wang Mang's rule and his standards only magnifies this display. The bronze *yue* ladle

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[387] Sivin, "Cosmos and Computation," 4.

shows that these acts of control may have travelled beyond the court, all the while relating the basis of the standardized weights and measures back to the measure of *Huangzhong*. This is evident in the minor inscriptions and the façade inscription on each of the artifacts respectively, as well as in the *Han shu* LLZ.

In reality, these tools constructed a fantasy of control anchored in a theory of cosmological resonance. The current evidence shows that the weights and measures remained generally steady as an imperial standard from the late Warring States down to the Eastern Han, making any act of standardization by an emperor either a rhetorical device upon taking the reins of authority, or an act of re-calibration of existing measures that did not change much from court to court, but possibly did across the imperial territories. The administration that emperors, officials, and experts built in the Western Han and Xin dynasties mainly affected their immediate environment and regional headquarters. Outside the court, things were quite different. Communities in different locations across the territory continued using their own metrological standards. Lack of oversight as well as individual interests caused much variance in sizes. People outside the court had no reason to care about possible consequences of badly tuned pitch pipes or an out-of-sync cosmos, unless they had direct political ties to the court.

Nevertheless, the extant evidence points to the conclusion that the process by which Liu Xin used the dimensions of a *Huangzhong* pipe to create a *yue* vessel and compare between the two by using millet grains did take place. It may have been a one-time ritual act, not repeated elsewhere, or it may have been used when creating models for copying. This information is not available to us yet. And still, the most important missing evidence to tie them together is missing – there is no excavated *Huangzhong* pitch pipe. Even though several pitch pipes have been excavated over the years, none matched the size requirements of

Huangzhong. The pitch pipe set unearthed in Mawangdui, was a burial artifact, not meant to be used.[388] A single pitch pipe measuring 7.76cm named the *Wuyi* pitch pipe is stored in the Shanghai museum. One end of the pipe is broken. It was passed down, and its origins are uncertain, although its inscription dates it to Wang Mang's era.[389] The two jade cylinders unearthed from the tomb of Liu He are awaiting further analysis.

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[388] He Jiejun 何介鈞, *Mawangdui Han mu* 馬王堆漢墓 (Beijing: *Wenwu chubanshe* 文物出版社, 2004).

[389] Wang Zichu 王子初, *Zhongguo yinyue kaogu xue* 中國音樂考古學, *Zhongguo chuantong yinyuexue congshu* (Fuzhou: *Fuzhou Jiaoyu* 福建教育, 2004), 341–43.

CONCLUSION

This dissertation was motivated by my larger quest to discover different ways to think about sound. My goal in this work was to explore how thinkers conceptualized sound in early China. I first sought to understand the components that made up early Chinese tuning theories and how they functioned and changed over time. Then, I sought to understand how administrations and thinkers used tuning theories when discussing the meaning and purpose of sound. In many cases these concepts were part of cosmological theories that aimed to "sum up hidden realities and subtle transformations" (*youyin zhi qing, jingwei zhi bian, ke de er zong ye* 幽隱之情，精微之變，可得而綜也).[390]

Cosmological thought attempts to explicate the workings of the world outside of human society by examining and deciphering patterns in Nature.[391] Early China had several cosmologies. In this dissertation, I focused on a cosmology of a computational nature. This type of cosmology was popular in China's political culture from around the mid-third century B.C.E., and especially flourished between the first century B.C.E. and the first century C.E., under the Western Han and Xin courts. Thinkers who used this cosmological approach relied on numbers and measurements as tools to express their ideas. Their purpose was to explain and provide ways of measuring the regularities of processes which were otherwise not so easily explained. These included the temporal and spatial rise and fall of *yin* and *yang*,

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[390] *Houhan shu* 後漢書, 2999.

[391] I use "Nature" similarly to the way Nathan Sivin defined it, as a "convenient synonym for "cosmos" that more obviously include terrestrial phenomena." Nathan Sivin, "State, Cosmos, and Body in the Last Three Centuries B.C," *Harvard Journal of Asiatic Studies* 55, no. 1 (1995): 5.

the function of *qi*, and the resonance between Nature and the realm of man. A great deal of this cosmological approach relied on hexagram interpretations and numerological patterns from the *Book of Changes*. Numbers were not simply counters, but regularities that embodied patterns of change. In a cosmology of this kind, sound was an agent of *yin-yang*, the five phases 五行, *qi*, and resonance. Its essential quality of existing in time, somewhere between the tangible and the intangible added to its appeal.

This dissertation highlighted four activities between the third century B.C.E. and the first century C.E. of which cosmological tuning theory was a main feature: ritual music performance, hexagram divination, calendric computations, and the standardization of weights and measures. Cosmological tuning theory also included creation myths, such as the tale of Ling Lun, and it may have been used in name divination, as seen in the case of Jing Fang's name change.

The question on the relationship between theory and practice remains difficult, but textual and archeological evidence attest that tuning theories were used in practice, at least in some cases. Scroll B of the excavated Fangmatan day books shows that members of elite society used tuning standards as part of a divination system for their daily activities. The "Da Siyue" 大司樂 section in the *Rites of Zhou* shows how tuning standards and scales were categorized and used in tunes and songs for ritual music performance. The *zhun* 準 string instrument, constructed by Jing Fang, its accompanying calculations, and the practice of Observing Qi, show that Jing Fang had intended tuning standards to be measured, marked and used, and not remain mere abstraction. Lastly, the *yue* volume measurement, discussed in the *Han shu* LLZ and seen in the *Jia Liang Hu* vessel asserts that the *Huangzhong* pitch pipe played an important role in Liu Xin's standardization of weights and measures during Wang Mang's reign.

In early China, sound was a technology. It could be audible or inaudible. It could be sung and played, and it could be calculated and measured. Ritual musical performance certainly used it, but so did an array of other activities, which modern scholars may think of as non-musical settings. For thinkers in early China, sound was a resonating *qi* emanating from the cosmos. Its calculation, manipulation, categorization and measuring were central to the synchronization between macrocosm and the political microcosm. Rulers used it to demonstrate their ability to be in tune with the cosmos, thus validating their mandate and power.

Paths for Future Research

Joseph Lam termed the act of writing about music theory, tuning theory, music, musical performance, and any activity related to sound as part of a discourse between thinkers as Musiking.[392] He provides examples specific to the Ming court, but his concept applies to thinkers like Jing Fang, as well as the experts gathered by Liu Xin to write the treatise on tuning and astronomy, and those who used sound in divination. These are also acts of Musiking. Writing about sound was not unique to officials in the music or astronomy bureaus. The culture in which these interactions flourished deserves broader inquiry.

Among other jobs still left to do is reexamine treatises on music and ritual, such as the "Treatise on Ritual and Music Performance" ("Liyue zhi" 禮樂志) in the *Han shu* and the treatise on "Ritual and Etiquette" ("Liyi" 禮儀) in the *HHS*, in light of the conclusions of this dissertation. This may help us to better understand the importance of cosmic tuning,

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[392] Joseph S. C. Lam, "Music And Male Bonding In Ming China," in *Male Friendship in Ming China* (Leiden, The Netherlands: Brill, 2007).

and how and where it was employed in connection with ritual music performance. Are the information and ideas in these treatises consistent with that in writings on tuning and mathematical astronomy? My preliminary hypothesis is that they are. Although the *Han shu* "Treatise on Ritual and Music Performance" discusses tuning theory in less detail than the *Han shu* LLZ, it still has much to offer and it does not disregard the cosmological aspect of sound. The *HHS* "Treatise on Ritual and Etiquette" has more concrete examples of how officials used tuning standards in rituals, such as the solstitial rituals mentioned in this dissertation.

Another important source for concepts of sound and tuning is the *Huainanzi* 淮南子. Chapters such as "Celestial Patterns" ("Tianwen" 天文) employ similar terminology and the same tuning theory as the *HHS* LLZ, and ideas that actually originated in the *Lüshi Chunqiu* 呂氏春秋 chapters on sound. Comparing these sources through their use of tuning could produce interesting results, and add to our knowledge of cosmological sound, and of the concept of resonance that is so strikingly depicted in the *Huainanzi*. The *Huainanzi* is also a rich source for various opinions, at times contradictory and at times syncretic, about the correct way to employ sound, and the sensorial reaction of the sage to natural sound and ritual music, among others. Some of these opinions reflect the court's ideology regarding sound, and the regulations of ritual music performances, while others disdained and mocked it, because it was man-made rather than natural and therefore unnecessary, or because it was licentious, extravagant, assaulting the senses, and ill motivated, thus missing the purpose of music, as they defined it, altogether.[393]

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[393] For a recent analysis of musical references in the *Huainanzi* see Avital H. Rom, "Echoing Rulership—Understanding Musical References in the Huainanzi," *Early China* 40 (2017): 125–65.

The subject of aural sensations in a ritual setting relates directly to ideas about the effects of music on the listener, the correct purpose of using sound. One was to bring about conformity to social norms. Another such aspiration was to reach a mental state of equanimity (*le* 樂 or *ping* 平), which required a connoisseurship of sound. This goal was often recommended in late Warring States and Han texts about music. The methods of training one's senses in order to reach this state varied despite their aim to achieve the same result. Exploring how ideas about sound and tuning may have played a part in the path to the equanimity of the mind will help bridge the gap between the technical, psychological, and cosmological.

LIST OF ABBREVIATIONS

Han shu	*History of the Former Han* (*Han shu* 漢書)
HHS	*History of the Later Han* (*Houhan shu* 後漢書)
LLZ	Treatise on Tuning Standards and Mathematical Astronomy (Lüli zhi 律歷志)
FMTB	Scroll B of the Fangmatan daybooks (*Fangmatan yi* 放馬談乙)

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